# Evidence for base-driven alternation in Tgdaya Seediq

**Abstract.** Standard morphophonological analysis allows **composite** URs, which "cobble" together information from multiple slots of a paradigm (Kenstowicz & Kisseberth, 1977). In contrast, under the **single surface base hypothesis** (Albright 2002 *et seq.*), the input to morphophonology must be a single slot in a paradigm. In this paper, I compare the two approaches by examining verb paradigms in Tgdaya Seediq. In a corpus study of the Seediq lexicon, I find that isolation stems are much more informative than suffixed forms. This asymmetry is argued to support the surface-base approach. Results are further supported in an experiment, where speakers productively extended alternations from the isolation stem to novel suffixed forms. Interestingly, speakers over-generalised certain patterns instead of matching lexical statistics in the experiment. I propose that this non-viridical learning is the result of a complexity bias, which cannot be accounted for in existing surface-base models of morphophonological learning. Instead, I propose a constraint-based analysis of Seediq alternations, with a complexity learning bias.

## 1 Introduction: two approaches to morphophonology

The classical approach to morphophonological analysis, laid out by Kenstowicz & Kisseberth (1977), involves setting up underlying forms (URs) which preserve as many contrastive phonological properties as possible. When all forms of a paradigm are affected by neutralization, the resulting UR must combine information from multiple slots of a paradigm, and is in this sense 'composite'. For example, in Tonkawa verbal paradigms, verb roots display extensive morphophonemic alternations as illustrated in (1). Different vowels of the verb stem surface depending on the phonological properties of its affixes. Crucially, for trisyllabic stems like the ones in (1), no surface form has all three vowels. Instead, URs must combine information about the first vowel from slots 'A' or 'C' of the paradigm, and information about other vowels from other slots (e.g. slot 'D') (Kenstowicz & Kisseberth, 1977, 33). Under this approach, the UR can only be found by looking at multiple forms of a paradigm, and might not correspond directly to any single existing surface form.

(1) *Verbal alternations in Tonkawa* (Hoijer, 1946, cited by Noske, 2011)

| A | B | C | D | | |
|---|---|---|---|---|---|
| **notx**-oʔ | we-**ntox**-oʔ | **notxo**-n-oʔ | we-**ntoxo**-n-oʔ | 'hoe' | /notoxo/ |
| **netl**-oʔ | we-**ntal**-oʔ | **netle**-n-oʔ | we-**ntale**-n-oʔ | 'lick' | /netale/ |
| **picn**-oʔ | we-**pcen**-oʔ | **picna**-n-oʔ | we-**pcena**-n-oʔ | 'cut' | /picena/ |

Albright (2002b, et seq.) proposes an alternative approach, called the *single surface base hypothesis*, where the UR must be based on a single surface form in the paradigm. A slot in the paradigm is selected as a 'privileged base'. This base form is constrained to be the same slot of paradigm for all lexical items of a given category, and serves as the input for morphophonology.

In the Tonkawa example, the input to morphophonology would therefore have to be one of slots A-D of the paradigm. Under this approach, the grammar will have fewer informational resources available and be more prone to exceptions, as no allomorph can perfectly predict all three vowels of a verb stem. For example, if slot D were chosen to be the base, Tonkawa speakers would have to memorize the fact that in slots A and C of the paradigm, 'hoe' surfaces with the initial vowel [o], while 'lick' surfaces with the vowel [e]. Despite this limitation, the process of UR building is less complex and more restrictive, as there is no need to reference multiple slots of a paradigm.

In Tgdaya Seediq (henceforth Seediq), processes of vowel and word-final consonant neutralization cause all forms of a paradigm to suffer loss of contrasts, making it a good test case for comparing the two theories of morphophonology. As a preview, I find evidence in Seediq verbal paradigms that supports Albright's single surface base hypothesis. First, one slot of the Seediq paradigm is far more informative than the other slots, suggesting that reanalyses from a base slot (i.e. the more informative slot) has occurred. Additionally, Seediq stems have asymmetries in the distribution of segments that would be arbitrary under a composite UR approach, but are predicted by base-driven reanalysis. Finally, in an experiment, Seediq speakers were found to productively extend generalizations from the base form.

Notably, however, Seediq participants also extended alternations from stem form non-viridically, and overextended a pattern beyond environments predicted by the lexicon. I argue that this is the result of a complexity bias, where speakers preferentially learn simpler generalizations, rather than ones that apply to narrowly defined environments. Existing surface-base models of morphophonological learning, such as the Minimal Generalization Learner (MGL; Albright, 2002b; Albright & Hayes, 2003), are not able to account for this kind of learning bias. As a solution, I propose an alternative constraint-based analysis of the Seediq data, where the input is the designated base form, rather than composite URs. This model is implemented inn Maximum Entropy Harmonic Grammar (Goldwater & Johnson, 2003), which is a probabilistic variant of Optimality Theory (Smolensky, 1986; Prince & Smolensky, 1993). A learning bias is implemented as a prior probability distribution on constraints (Wilson, 2006).

## 2 Phonological alternations in Seediq verbal paradigms

Seediq is an Austronesian (Atayalic) language spoken in Central and Eastern Taiwan. The Tgdaya dialect, which the current study focuses on, is spoken primarily in Nantou. Although there are around 6500 Seediq people living in Nantou census2020, the number of fluent speakers is thought to be much fewer than this, due to high rates of language attrition.

The Seediq phoneme inventory is given in (2) and (3); where the orthography that I adopt differs from standard IPA, phonetic transcription is given in brackets. Seediq verbs are almost always inflected for voice, mood, and aspect; verbal inflection can take the form of prefixes, infixes or suffixes (Holmer, 1996). The Seediq inflectional affixes are summarised in Table 1. As will be described in the rest of this section, distributional restrictions cause there to be extensive vowel and consonant alternations between the unsuffixed and suffixed forms of a verb paradigm.

(2) *Seediq consonant inventory*

| | | | | | | |
|---|---|---|---|---|---|---|
| Stops | *p b* | *t d* | | *k g* | *q* | ʔ |
| Fricatives | | *s* | | *x* | | *h* |
| Affricates | | *c* [t͡s] | | | | |
| Nasals | *m* | *n* | | ŋ | | |
| Approximants | | *r* [ɾ] | *y* [j] | *w* | | |
| Laterals | | l | | | | |

(3) *Seediq vowel inventory*

| | | |
|---|---|---|
| i | | u |
| e | | o |
| | a | |

|  | AGENT FOCUS | LOCATIVE FOCUS | PATIENT FOCUS | INSTRU. FOCUS |
|---|---|---|---|---|
| PRES | -m-/mu- | -an | -un | su- |
| PRET | -mun- | -n-, -an | -un- |  |
| FUT | mu(pu)- | RED-an | RED-un |  |
| IMP |  | -an-i | -i |  |

Table 1: Inflectional morphology of Seediq

## 2.1 Data collection

The descriptive generalizations to be outlined in the rest of this section are all taken from Yang (1976), supplemented with counts from the current study. In particular, I confirm these generalizations using a corpus of 341 verbal paradigms. Paradigms were drawn from (1) the Taiwan Aboriginal e-Dictionary (Council of Indigenous Peoples, 2020), and (2) fieldwork with three Seediq speakers (ages 69-78), carried out by the author in Puli Township, Nantou, Taiwan. Data was collected over the course of three weeks in July 2019. There is a high rate of language attrition in Seediq communities, such that fluent speakers are mostly above age 40, and only speakers around age 60 and above consistently use Seediq in daily conversation. As such, the speakers consulted in this study likely represent a more conservative variant of Seediq. All three consultants reported speaking Mandarin and Seediq regularly at roughly equal rates.

188 paradigms were collected from the online dictionary, and the remaining 156 paradigms were collected from native speaker consultants. Verb paradigms taken from the dictionary were confirmed with consultants, and omitted if my consultant(s) did not recognise the word, or provided conflicting inflected forms. Three forms were omitted under these criteria, leaving a total of 341 paradigms to be analyzed. The omitted words are given in (4); (4a) was not recognised by my consultants, and for the other words, consultants disagreed with the dictionary on the suffixed form. For the newly collected forms, there was a high degree of inter-speaker agreement; unless otherwise specified, all three consultants agreed on the forms collected.

(4) *Discrepancies in dictionary and consultant responses*

|  |  | STEM | SUFFIXED |  |
|---|---|---|---|---|
|  |  |  | *dict.* | *consultant* |
| (a) | ‘to hook’ | ˈdaquc | duˈqut-an | NA |
| (b) | ‘to increase’ | ˈuman | ˈmal-an | ˈman-an |
| (c) | ‘to seal/close’ | ˈsepuy | suˈpuy-an | suˈpuw-an |

Note that all verbs were elicited with the /su-/, /-an/, /-un/, and /-i/ affixes. Because the patterns reported in the paper were found to be consistent across affixes, all examples (unless otherwise specified) will only compare the bare stem forms (which are representative of all unsuffixed slots of the paradigm) to forms suffixed with /-an/ ‘LOCATIVE FOCUS.PRES.’ (which are representative of all suffixed slots).

## 2.2 Stress-driven vowel alternations

Seediq stress is always penultimate; suffixation shifts stress rightwards (Yang, 1976), giving rise to alternations such as [ˈ**bu**nuh]∼[buˈ**nu**han] ‘wear hat’. Crucially, stress interacts with vowel quality, resulting in vowel alternations between the stem and suffixed forms of the paradigm.

Pretonically, all vowel contrasts are neutralised; in the following examples, the neutralized segment in each paradigm is highlighted in grey. First, onsetless pretonic vowels are deleted, as illustrated in (5). The pretonic vowel will assimilate to an adjacent stressed vowel if the two are separated by [ʔ] or [h] (see (6)). Otherwise, vowels are reduced to [u] pretonically, as in (7). This last process of reduction to [u] is by far the most common, occurring in 276 stems. All three pretonic vowel neutralization processes are exceptionless.

(5) *Onsetless vowels delete* (35/35)

| | STEM | SUFFIX | COMP. UR | GLOSS |
|---|---|---|---|---|
| (a) | ˈawak | ∅ˈwak-an | /awak/ | ‘lead (by a leash) |
| (b) | ˈeyah | ∅ˈyah-an | /eyah/ | ‘come’ |
| (c) | ˈuyas | ∅ˈyas-an | /uyas/ | ‘sing’ |

(6) *Vowel assimilation across [ʔ] or [*h*]* (25/25)

| | | | | |
|---|---|---|---|---|
| (a) | ˈleʔiŋ | liˈʔiŋ-an | /leʔiŋ/ | ‘hide (an object)’ |
| (b) | ˈsaʔis | siˈʔis-an | /saʔis/ | ‘sew’ |

(7) *Vowel reduction to [u]* (276/276)

| | | | | |
|---|---|---|---|---|
| (a) | ˈgedaŋ | guˈda-an | /gedaŋ/ | ‘die’ |
| (b) | ˈbiciq | buˈciq-an | /biciq/ | ‘decrease’ |
| (c) | ˈbarah | buˈrah-an | /barah/ | ‘rare’ |
| (d) | ˈburah | buˈrah-an | /burah/ | ‘new, create’ |

Pretonic vowel neutralization always results in a loss of contrasts in the *suffixed* forms. For example, consider examples (7c-d). The two words are distinctive in the isolation stem, but homophonous in the suffixed form due to reduction of the stem's initial vowel.

Post-tonically, similar but more restricted processes of vowel reduction are observed, where /e, o, u/ reduce to [u] in post-tonic closed syllables. This results in alternations where a post-tonic [u] in the stem form may surface as [e], [o] or [u] when stressed in the suffixed form. Examples of such alternations are given in (8).

(8) *Post-tonic reduction of /e,o/ to [u]*

| | STEM | SUFFIXED | UR | GLOSS | |
|---|---|---|---|---|---|
| (a) | ˈremux | ruˈmuxan | /remex/ | ‘enter’ | (u∼u, n=60) |
| (b) | ˈpemux | puˈmexan | /pemex/ | ‘hold’ | (u∼e , n=36) |
| (c) | ˈdoʔus | doˈʔos-an | /doʔos/ | ‘refine’ (metal)’ | (u∼o, n=4) |

In addition, with the exception of /uy/, diphthongs are prohibited in post-tonic (i.e. word-final) position. /ay/ and /aw/ are respectively monophthongized to [e] and [o] as in (9a-b), while /ey/ is monophthongized to [u] as in (9c).

(9) *Word-final monophthongization*

| | STEM | SUFFIXED | UR | GLOSS | |
|---|---|---|---|---|---|
| (a) | ˈraŋe | ruˈŋay-an | /ranay/ | ‘play’ | (e∼ay, n=7) |
| (b) | ˈsino | suˈnaw-an | /sinaw/ | ‘to drink (alcohol) | (o∼aw,n=1) |
| (c) | ˈdeŋu | duˈŋey-an | /deŋey/ | ‘to dry (food)’ | (u∼ey, n=12) |
| (d) | ˈseku | suˈkuw-an | /seku/ | ‘to store’ | (u∼u, n=13) |

Stem-final /Vg/ also neutralizes to vowels, resulting in final vowel alternations. As summarised in (10), /ag/ neutralizes with [o] word-finally (10a), /eg, ug/ both neutralize to [u] (10b), and /ig/ becomes [uy] (10c). These alternations are historically a result of /g/ weakening to [w] word-finally, followed by monophthongization of the resulting diphthong (Li, 1981).

(10) *Alternation of final /g/*

|  | STEM | SUFFIXED | UR | GLOSS |  |
|---|---|---|---|---|---|
| (a) | ˈhilo | huˈlag-an | /hilag/ | ‘cover’ | (o∼ag, n=9) |
| (b) | ˈlihu | luˈhug-an | /hilug/ | ‘string together’ | (u∼ug/eg, n=9) |
| (c) | ˈbaruy | buˈrig-an | /burig/ | ‘buy/sell’ | (uy∼ig, n=3) |

Stem-final [e] only results from monophthongization of /ay/, so final [e] predictably alternates with [ay] in the suffixed form, barring a small subset of irregular alternations. On the other hand, the final [u] of an unsuffixed form could alternate with [ey], [ug], or [eg]. There are also stems where final [u] is non-alternating, as in (9d). Note that in this example, [w] is also inserted to resolve vowel hiatus; hiatus resolution by either glide or glottal stop insertion is a regular and predictable process in Seediq. In other words, the final [u] of an unsuffixed form has multiple possible alternants in the unsuffixed form. Similarly, stem-final [o] has multiple possible alternants in the suffixed form, and can alternate either either [aw] or [ag].[1]

In general, post-tonic neutralizations result in a loss of contrasts in the isolation stem, so that it is not possible to predict how a final vowel will alternate from just the stem. For example, the stem-final vowels of (8a) and (8b) are contrastive in the suffixed form, but both reduce to [u] in the isolation stem.

## 2.3 Final consonant alternations

In addition to the vowel neutralization processes described so far, Seediq has phonotactic constraints against word-final [p b m t d l g], motivating various processes of word-final neutralization. /p, b, m, t, d, l/ are neutralised with other consonants as outlined in (11).

(11) *Processes of final consonant alternations*

(a) /p/, /b/, /k/ → [k]
(b) /d/, /t/, /c/ → [c]
(c) /m/, /ŋ/ → [ŋ]
(d) /l/,/n/ → [n]

As a result of (11a), the final [k] of a stem could surface as [k] in the suffixed form, or alternate with either [p] or [b]. Examples of each possibility are provided in (12a-c). In (12d-j), similar examples are provided for the other final consonant alternations.

As will be discussed further in §3, rates of alternation differ depending on the identity of the final consonant. For example, stem-final [ŋ] tends not to alternate; [ŋ]∼[m] alternation is only observed in three out of 35 ŋ-final forms (12h). In contrast, stem-final [c] almost always alternates with [t] (12e).

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[1] Note that [o] has a limited distribution in Seediq; it surfaces post-tonically as the result of post-tonic neutralization, but there are very few stems that su.rface with phonemic stressed [o] as in (8c). In the current data, only four were found.

(12) *Alternation of final /p, b, m, t, d, l/*

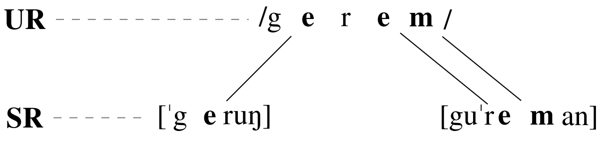
|  | STEM | SUFFIXED | UR | GLOSS |  |
|---|---|---|---|---|---|
| (a) | ˈtatak | tuˈtak-an | /tatak/ | ‘chop’ | (k∼k, n=19) |
| (b) | ˈpatak | puˈtap-an | /patap/ | ‘cut’ | (k∼p, n=6) |
| (c) | ˈeluk | ∅ˈleb-an | /eleb/ | ‘close’ | (k∼b, n=1) |
| (d) | buˈcebac | bucuˈbac-an | /bucebac/ | ‘slice’ | (c∼c, n=1) |
| (e) | ˈdamac | duˈmat-an | /damat/ | ‘for eating’ | (c∼t, n=16) |
| (f) | ˈharac | huˈrad-an | /harad/ | ‘build (a wall)’ | (c∼d, n=4) |
| (g) | ˈgilaŋ | guˈlaŋ-an | /gilaŋ/ | ‘mill (rice)’ | (ŋ∼ŋ,n=32) |
| (h) | ˈtalaŋ | tuˈlam-an | /talam/ | ‘run’ | (ŋ∼m, n=3) |
| (i) | ˈdurun | duˈrun-an | /durun/ | ‘entrust’ | (n∼n, n=3) |
| (j) | ˈdudun | duˈdul-an | /dudul/ | ‘lead’ | (n∼l, n=19) |

## 2.4 Two approaches to morphophonology in Seediq

Aas a result of vowel reduction and word-final consonant neutralization, Seediq verbs undergo extensive alternations, and all forms of a Seediq verbal paradigm suffer from some form of neutralization. This complicates the task of analyzing Seediq verbal paradigms, and poses a potential challenge for Seediq learners. This is because, when given just one form of a paradigm (either an unsuffixed or suffixed form), there is no way to perfectly predict the other slots of the paradigm.

The standard approach to dealing with this issue, which was taken up in Yang’s (1976) analysis of Seediq, is to use composite URs. Specifically, URs are set up by cobbling information from the unsuffixed forms (which are not affected by pretonic vowel neutralization) and the suffixed forms (which are not affected by post-tonic neutralizations). For example, consider the verb [ˈgeruŋ]∼[guˈreman] ‘to break’. Given this paradigm, the learner can construct a UR /gerem/ which takes its initial vowel from the unsuffixed form, and its final vowel and consonant from the suffixed form; this is illustrated in (13).

(13) *Composite UR approach for ‘to break’*



Under a composite UR approach, the majority of forms in Seediq (excluding a subset of irregularly alternating forms) can be derived using standard, phonotactically motivated markedness constraints. Example (14) demonstrates this in Optimality Theory (Prince & Smolensky, 1993), for the UR /gerem/.

A highly ranked markedness constraint $*\text{m}]_{\text{w}}$, suitably ranked against Faithfulness, rules out candidates (a)-(b). Post-tonic vowel reduction to [u] is enforced by a positional licensing constraint, Lic(nonperiph/stress), which limits non-peripheral vowel qualities to stressed syllables (Crosswhite, 2004). This constraint rules out candidate (c), where the vowel surfaces faithfully as [e] in unstressed position. For now, competing candidates are not shown, and the specific patterns of alternation observed (e.g. /m/→[ŋ]) are assumed to result from the interaction of faithfulness constraints.

(14) *Derivation of* [ˈgeruŋ] *under a composite UR approach*

| /gerem/ | *m]$_W$ | Lic-nonPer | ID-LAB | ID[back] |
|---|---|---|---|---|
| a. ˈgerem | ∗! | ∗ | | |
| b. ˈgerum | ∗! | | | ∗ |
| c. ˈgereŋ | | ∗! | ∗ | |
| ☞ d. ˈgeruŋ | | | ∗ | ∗ |

For now, the key point to note is that these constraints hold true across the entire Seediq lexicon, with few to no exceptions. As such, although UR discovery is more complex under the composite UR approach, the resulting grammar is elegant and relatively simple. Moreover, this approach makes empirically testable predictions about the range of possible alternations in Seediq.

In contrast, under the surface-base approach to UR construction, the Seediq learner would designate a surface allomorph to be the base (which is constrained to a single slot of the paradigm). In the case of Seediq, this means that the base would have to be either the unsuffixed form for all verbs, or the suffixed form for all verbs.

The resulting grammar is more complicated because the base, whether it is the suffixed or unsuffixed form, suffers from neutralization. For example, if the isolation stem (unsuffixed) form were the base, the grammar would need to somehow ‘undo’ final consonant neutralization, which is impossible do achieve with perfect accuracy. As a result, any constraints (or rules) in the grammar will have exceptions which must be dealt with through methods like diacritics or lexical listing, or other stipulations that are not motivated by general markedness.

For example, consider again the stem-suffix pair [ˈgeruŋ] ∼ [guˈreman]. Assuming that [ˈgeruŋ] is the base, the grammar would need a constraint *ŋV (no prevocalic [ŋ] ) to enforce [ŋ]∼[m] alternation. This is demontrated in tableau (15). However, this constraint would predict the wrong output for a stem-suffix pair such as [ˈgilaŋ]∼[guˈlaŋan], where [ŋ] is non-alternating.

(15) *Derivation of* [guˈreman] *under a single surface-base approach*

| [ˈgeruŋ]-an | *ŋV | Faith |
|---|---|---|
| a. guˈreŋan | ∗! | |
| ☞ b. guˈreman | | ∗ |

On the other hand, as noted in §1, UR discovery under the single surface base hypothesis is relatively easier. There is also increasing evidence in support of the single surface-base hypothesis from from various sources, including historical change in languages like Korean (Kang, 2006) and Yiddish (Albright, 2010). This historical evidence is further supported by results of wug tests (Jun, 2010) and surveys of child errors (Kang, 2006) for Korean.

Both the composite UR and surface-base approaches are able to account for the Seediq data (with relative strengths and weaknesses). We can compare the two approaches by examining their predictions about the type of mislearning that happens. Broadly speaking, the composite UR approach predicts that when the learner has incomplete data (resulting in reanalysis of paradigms), the UR will be determined on the basis of whatever surface forms are available. For example, assuming a straightforward mechanism where the language learner simply takes whatever surface form they hear to be the UR, they might posit the UR /geruŋ/ from the unsuffixed [ˈgeruŋ], and project the suffixed form [guˈruŋan]. On the other hand, if the learner hears the suffixed form [guˈreman], they might posit the UR /gurem/, and infer that the isolation stem is [ˈguruŋ]. Language learners may also utilize a more sophisticated approach, such as by forming the UR on the basis of relevant lexical frequencies (Jun, 2010). Regardless of the learner's strategy, reanalyses in both directions are plausible, and the resulting Seediq lexicon should reflect this.

The surface base approach makes markedly different predictions compared to the composite UR approach with respect to how a learner behaves given incomplete data. Namely, reanalyses will always be projected from the designated base. This predicts that the resulting Seediq lexicon will have asymmetries in paradigm structure, reflecting asymmetries in reanalysis.

## 3 Stem-suffix asymmetries in Seediq

In this section, I use the Minimal Generalization Learner algorithm (MGL; Albright, 2002b; Albright & Hayes, 2003) to compare the informativeness of Seediq isolation stems and suffixed forms. The MGL is a surface-base model which learns surface mappings between inflected forms. It takes as its training data a set of pairs of morphologically related surface forms (in this case stem and suffixed forms), and attempts to learn the set of stochastic morphological mappings that project one from the other. I use the MGL to learn both a grammar that maps from isolation stem to suffixed forms, and one that maps from suffixed to stem forms. The two models are then assessed for how accurately they predict the Seediq lexicon.

As a preview, although neither the stem nor suffixed forms can perfectly predict the rest of the verb paradigm, statistical tendencies in the data make it so that stem forms are much more informative than suffixed forms. In other words, suffixed forms are highly predictable from stems, but the stems are not as predictable from the suffixed forms.

This asymmetry supports the single surface-base approach. This is because, under a system where speakers have selected one cell in the paradigm to be a base, verb paradigms whose other cells are poorly predicted by the base will be gradually leveled. This process acts as a feedback loop, in that reanalyses will continue to increase the informativeness of the base forms. If one cell in a paradigm is much more informative than the other, and this asymmetry cannot be attributed just to phonological neutralization processes (e.g. vowel reduction), restructuring from a single base form has likely happened.

For Seediq, the stem-suffix asymmetry suggests that speakers have designated the stem form to be the base, and that restructuring over time has exaggerated statistical tendencies which cause the stem base to be much more informative than the suffixed forms of the paradigm.

Note that, as described in §2, Seediq verbal paradigms have prefixed forms. Because the isolation stems and most prefixed forms show the same patterns of neutralization, there is no way to differentiate between stem and prefixed forms in terms of their suitability as bases. The data presented will use the ISOLATION STEM form to represent all unsuffixed slots of a paradigm, but in principle, any unsuffixed slot could be the base.

The core argument of the single surface base hypothesis is that surface forms serve as the input to morphophonology. This theory of surface bases has primarily been implemented using the MGL-based model, but is in fact compatible with other theories of morphophonology. The model employed in this section is not meant to be a theoretical model, but rather a model of quantitative assessment, for testing whether the stem-suffix asymmetry exists in Seediq. In fact, as will be discussed in §5, in a wug test, Seediq speakers extended vowel copying beyond the environments predicted by a model of only morphological correspondences. This suggests that a theoretical model of Seediq alternations needs additional mechanisms.

### 3.1 The MGL algorithm

The MGL parses each stem-suffix pair, and attempts to learn a grammar that predicts which change each form will take. It does so by comparing forms that share the same change, discovering what phonological features they have in common, and generalizing rules based on shared features. The model is minimal in that it will retain specific rules, and only generalize more broadly defined rules when segments can be grouped using shared features. For details on the MGL, refer to Albright & Hayes (2003, p. 123-128).

For example, in learning stem→suffixed mappings, the model could compare pairs the two inputs in (16a) to learn a rule of suffixation after non-continuant dorsals. Including an additional input [ˈberas]∼[buˈrasan] would result in the model learning a more general rule (16b). Eventually, consideration of a broader range of forms would result in a general suffixation rule ∅ →**an**. To learn a grammar of the reverse mapping, from suffixed to stem forms, the model repeats this same algorithm with the suffixed allomorph as input.

(16) *Rule generalization in the MGL (stem-to-suffix mapping)*

| | **input** | **rule** |
|---|---|---|
| a. | ˈbeliŋ∼buˈliŋan, ˈbetaq∼buˈtaqan | ∅ → an / [+DORSAL,-continuant]_ |
| b. | ˈbeliŋ∼buˈliŋan, ˈbetaq∼buˈtaqan, ˈberas∼buˈrasan | ∅ → an / [-continuant]_ |

In stem-suffix pairs where neutralization results in ambiguity, the most general rule learned will not correctly predict all outputs. For example, neutralization of final [p] to [k] results in stem-suffix pairs like [ˈkayak]∼[kuˈyapan] ‘to cut’. The general stem-to-suffixed mapping of ∅→an would predict the wrong output *[kuyakan]. For such cases, the model learns minority patterns (e.g. k→pan), which exist alongside and compete with the more general rule.

Each rule is assessed for its *accuracy* (proportion of relevant forms correctly predicted by the rule). Accuracy values are then adjusted downwards using lower confidence limit statistics, such that rules with fewer data points will be penalized (Mikheev, 1997). This adjusted value, called *confidence*, better captures the fact that rules with very little evidence tend to be less reliable (Albright, 2002b). Confidence determines the probability of a rule applying to each input.

The resulting grammar learned by the MGL is a system of competing rules that vary in generality, and are each assigned a confidence value. When the grammar is invoked to produce an inflected form, all applicable rules are tried, resulting in a set of output candidates, each given a confidence score. For example, recall that given a Seediq isolation stem ending in [c], the [c] could potentially alternate with [t] or [d]. Reflecting this ambiguity, the stem→suffixed grammar learns three rules that apply to the input [ˈbirac]. These rules, given in (17), produce three output candidates of varying confidence.

(17) *Examples of rules learned by the MGL in a stem→suffix mapping*

| **input** | **output** | **rule** | **confidence** |
|---|---|---|---|
| ˈbira**c** | buˈra**c**an | ∅ → an / _ | 0.65 |
| | buˈra**t**an | c → tan / _ | 0.67 |
| | buˈra**d**an | c → dan / _ | 0.13 |

## 3.2 Model evaluation and implementation

We can assess the relative informativeness of different mappings learned by the model, to see which slot of the paradigm is on average better at predicting the other slots. The **base**, under this approach, is the slot that yields grammars which are best able to predict the lexicon.

To compare the informativeness of Seediq stem and suffixed forms, I trained the model on the 341-word corpus. The informativeness of a paradigm slot was taken to be its accuracy in predicting the other slot. Specifically, for each word in a mapping, the generalization with the highest reliability was taken to be the model's prediction. These predictions were compared against to lexicon, to calculate the proportion of forms correctly predicted by the grammar.

Note that this implementation only uses TYPE frequency information, and ignores the relative token frequency of different stems. Due to lack of corpus data for Seediq, it was not possible to obtain data on token frequency. In any event, the literature suggests that when studying speakers' productive knowledge of morphophonological patterns, type frequency is the more relevant measure, and a better predictor of speakers' linguistic intuitions (Albright, 2002b; Bybee, 2003; Pierrehumbert et al., 2003; Edwards et al., 2004, etc.).

The stem-to-suffix mapping in Seediq has two largely independent processes of allomorphy: post-tonic vowel alternation and final consonant alternations. The MGL cannot straightforwardly deal with this because it assumes that there is exactly one morphological mapping for each stem-suffix pair, and cannot concurrently treat vowel and consonant alternations as independent processes. The algorithm also assumes strictly local contexts, and is not able to learn non-local conditioning, such as the vowel matching pattern observed in mapping from stem to suffixed forms. This issue could potentially be resolved by modifying the MGL to search for non-local environments, as in Albright and Hayes (2006).

I opt for a simpler approach, and resolve the above issues by training two separate grammars for each mapping; one "segmental tier" grammar and one "vowel tier" grammar (Hayes & Wilson, 2008). A stem-suffix mapping is judged as being correctly predicted by the model only if both the segmental grammar and vowel-tier grammar predict the correct output.

| | grammar | | |
|---|---|---|---|
| | segmental tier | vowel tier | overall |
| stem→suffix | 78.2% | 86.3% | 72.7% |
| suffix→stem | 90.0% | 48.3% | 41.3% |

Table 3: Model results: accuracy of mappings between Seediq stem and suffixed forms

## 3.3 Results: comparing the accuracy of different mappings

Table 3 compares the relative accuracy of the stem-to-suffix mapping and suffix-to-stem mapping; full details of model implementation and results are given in the Supplementary Materials.

Overall, the stem-to-suffix mapping is much more informative, accounting for 72.7% of the lexicon (vs. 41.3% in the suffix-to-stem mapping). The discrepancy between the two mappings is largely because pretonic neutralization renders the penultimate vowel of the stem unpredictable in the suffix-to-stem mapping. In fact, the vowel tier model is only 41.3% accurate in the suffix-to-stem mapping. In Section 3.4 and Section 3.5, I detail why the suffixed form is much less informative, even though the isolation stem undergoes as much, if not more, phonological neutralization processes.

## 3.4 Predictability from the isolation stem

The Seediq isolation stem has three environments where neutralization has resulted in unpredictability: (i) post-tonic [u] in closed syllables (due to post-tonic reduction of mid vowels, (ii) [c, n, k, ŋ] in stem-final position (due to final consonant neutralization), and (iii) [o, u, e] in stem-final position (due to monophthongization and final-[g] neutralization). Crucially, statistical regularities allow speakers to 'undo' the neutralizations in these environments with relatively high accuracy, meaning that the isolation stem is informative despite undergoing neutralization. In the interest of space, only the first two environments are discussed; for a discussion of final vowel alternations, the reader is referred to Kuo (2020).

### 3.4.1 Undoing post-tonic vowel reduction

/e, o/ are reduced to [u] in post-tonic closed syllables. As a result, the final [u] of a CVCuC stem can surface as [e], [o], or [u] in the suffixed form. Although it is not possible to perfectly predict what this post-tonic [u] will surface as in the suffixed form, it turns out that the vowel which surfaces is strongly correlated with the identity of the isolation stem's stressed vowel.

Specifically, there is a tendency for VOWEL MATCHING, where the stressed vowel of the suffixed form 'matches' the stressed vowel of the isolation stem. This pattern is illustrated in Fig. 1, which shows the distribution of stressed vowels in CVCuC stems. The x-axis shows the stressed vowel of the isolation stem, while the y-axis shows the stressed vowel of the suffixed allomorph. For example, the bottom-left cell, where the stressed vowel of the stem and suffixed forms are both [o], corresponds to stem-suffixed pairs such as [ˈpotus]∼[puˈtosan]. The top two rows of Fig. 1 show irregular alternations, where post-tonic [u] alternates with [a] or [i] (instead of [e] or [o]). For example, the stem-suffix pair [ˈhuruc]∼[huˈrid-an] 'to stop suddenly' contains an irregular post-tonic [u]∼[i] alternation.

Exceptions aside, if the stem stressed vowel is [o], the reduced [u] surfaces as [o] in the suffixed form (3/3, 100%). Similarly, if the stem stressed vowel is /u/, the reduced vowel will surface as [u] in the suffixed forms (29/31, 93%). For [e], there is similarly a strong tendency for vowel matching for around 79% of the relevant forms (34/43). Otherwise, if the stem stressed vowel is /a/ or /i/, the reduced vowel is usually non-alternating, and surfaces as [u].

The MGL is able to learn these tendencies, as summarized in Table 4 shows a subset of rules learned in the stem-base vowel tier grammar. Here, each rule predicts what the second vowel (V2) of a stem will surface as in the suffixed form. For ease of reading, rules have been schematized to more closely reflect Seediq surface forms, with the relevant vowel in bold. The grammar learns a general rule (R1) predicting non-alternation, but this rule is very low in confidence (0.44). Instead, R2 and R3, which predict vowel matching in CeCuC and CuCoC forms, are assigned higher confidence. In other words, the stem-base vowel

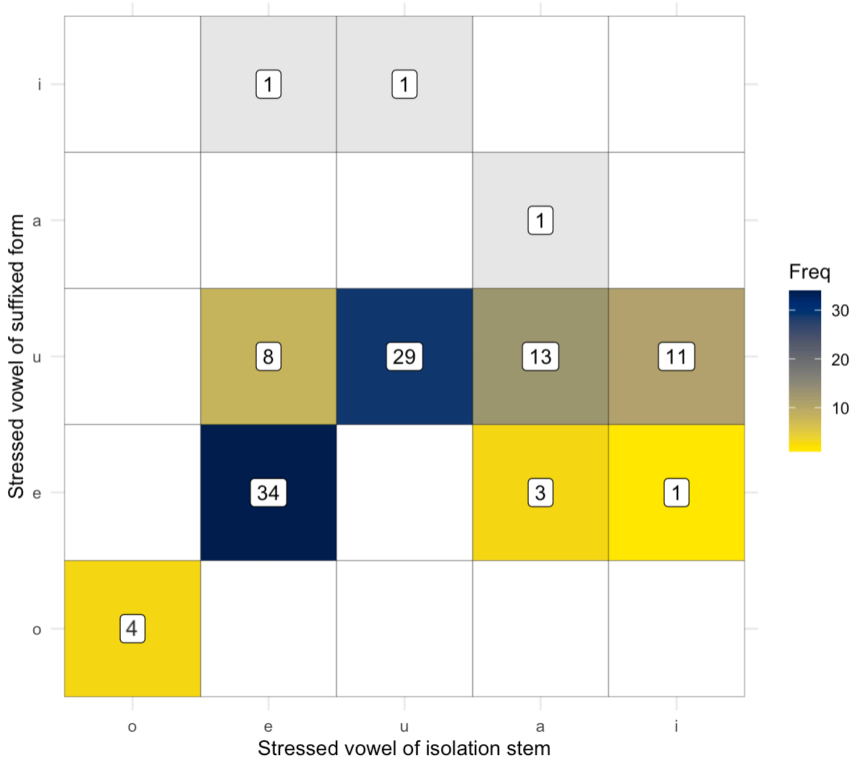


Figure 1: How the reduced [u] of unsuffixed CVCuC is realised when stressed under suffixation (Grey cells show irregularly alternating forms)

model predicts that speakers will extend [u]∼[e] and [u]∼[o] alternations in CeCuC and CoCuC stems. The grammar also learns that in CVCuC stems where V1 is not [e] or [o], the post-tonic vowel surfaces as [u] (R4 and R5).

| | rule | scope | hits | reliability | confidence |
|---|---|---|---|---|---|
| R1. | CVC**u**C→CuC**u**Can | 141 | 76 | 0.54 | 0.44 |
| R2. | CeC**u**C→CuC**e**Can | 60 | 46 | 0.77 | 0.72 |
| R3. | CeC**u**C→CuC**o**Can | 3 | 3 | 1.00 | 0.72 |
| R4. | CaC**u**C→CuC**u**Can | 28 | 19 | 0.68 | 0.61 |
| R5. | CaC**u**C→CuC**e**Can | 28 | 7 | 0.25 | 0.20 |

Table 4: Examples of rules learned in the vowel stem-base grammar

### 3.4.2 Undoing final consonant neutralization

Recall that word-finally, consonants /p, b, t, d, m, l/ are prohibited, resulting in the patterns of final consonant neutralization described in Section 2.3 and summarised in (18). As a result of these alternations, when given just the isolation stems, it is not possible to perfectly predict whether final [c, k, n, ŋ] will alternate in the suffixed form.

(18)

| STEM | | SUFFIXED |
|---|---|---|
| [c] | ∼ | [t, d, c] |
| [k] | ∼ | [p, b, k] |
| [n] | ∼ | [l, n] |
| [ŋ] | ∼ | [m, ŋ] |

However, final consonants tend to either almost always or almost never alternate, as summarised in Table 5. Final [ŋ] almost never alternates with [m]; the hypothetical stem [ˈpatiŋ] will surface as [puˈtiŋ-an] about 94% of the time. For final [k], rates of alternation are more intermediate, but there is still a tendency towards non-alternation, with 72% of stem-final [k] surfacing faithfully as [k] in the suffixed form.

In contrast, final [c] and [n] show a strong preference for alternation; final [c], in particular, alternates with either [t] or [d] 95% of the time. Only a single [c]-final stem in the data was found to be non-alternating. In addition, for stem-final [c] and [k], which each have two possible alternants, there is a strong tendency to alternate with the voiceless variant (e.g. [c] alternates with [t] more than with [d]).

| | **Cons.** | **Alternates?** | **Alternant** | **Example** | **Frequency** | |
|---|---|---|---|---|---|---|
| (a) | c | Yes | t | patic ∼ putitan | 16 | (76%) |
| | | Yes | d | patic ∼ putidan | 4 | (19%) |
| | | No | | patic ∼ putican | 1 | (5%) |
| (b) | n | Yes | l | patin ∼ putilan | 18 | (75%) |
| | | No | | patin ∼ putinan | 6 | (25%) |
| (c) | k | Yes | p | patik ∼ putipan | 6 | (23%) |
| | | Yes | b | patik ∼ putiban | 1 | (4%) |
| | | No | | patik ∼ putikan | 19 | (73%) |
| (d) | ŋ | Yes | ŋ | patiŋ ∼ putiman | 2 | (6%) |
| | | No | | patiŋ ∼ putiŋan | 33 | (94%) |

Table 5: Rates of final consonant alternation (irregularly alternating forms are excluded)

In general, because the distribution of final consonant alternants is very uneven, the alternant that surfaces in the suffixed form is actually highly predictable. This allows the MGL to learn an accurate stem→suffix mapping. Table 6 shows a subset of the rules learned in the stem-base segmental tier grammar, starting with a general suffixation rule (R1). The grammar learns high-confidence rules for [c]∼[t] alternation (R2) and [n]∼[l] alternation (R3). Additionally, the rules for [ŋ]∼[m], [c]∼[d], and [k]∼[p] alternation have very low confidence. This captures the intuition that given [ŋ]-final and [k]-final stems, the final consonant is most likely to not alternate.[2]

## 3.5 Predictability from the suffixed allomorph

Having discussed why the MGL learns a relatively accurate stem→suffix grammar, I now discuss why the opposite mapping, where the base is the suffixed form, is much less accurate.

All suffixed forms suffer from *pretonic* vowel neutralization. Because suffixation shifts stress rightwards, the penultimate vowel of all stems either reduce to [u], assimilate to the stressed vowel, or get deleted. In other

---

[2] By design, the MGL does not learn patterns that are observed in only one form. As a result, it does not learn the [k]-[b] alternation.

| | **rule** | **scope** | **hits** | **reliability** | **confidence** | **example** |
|---|---|---|---|---|---|---|
| R1. | ∅ →an | 341 | 230 | 0.67 | 0.65 | ˈpanah→puˈnahan |
| R2. | c→tan | 23 | 17 | 0.74 | 0.67 | ˈdamac→duˈmatan |
| R3. | n→l/i__ | 8 | 7 | 0.88 | 0.74 | ˈdudun→duˈdulan |
| R4. | ŋ→m | 35 | 2 | 0.06 | 0.04 | ˈtalaŋ→tuˈlaman |
| R5. | c→dan | 23 | 4 | 0.17 | 0.13 | ˈharac→huˈradan |
| R6. | k→pan | 28 | 7 | 0.25 | 0.20 | ˈpatak→puˈtapan |

Table 6: Examples of rules learned in the segmental stem-base grammar

words, given a suffixed form like [puˈtis-an], it is impossible to perfectly predict what the [u] will surface as under stress in the isolation stem.

In the case of post-tonic vowel alternations, a correlation between the stressed vowels of the stem and suffixed forms made it possible to 'undo' post-tonic vowel reduction with relatively high accuracy. For pre-tonic vowel reduction, however, there is less of a clear pattern of predictability in vowel distributions. This is demonstrated in Fig. 2, which shows the distribution of vowels across *all* disyllabic stems in the lexicon. The x-axis shows the stressed vowel of the suffixed form, while the y-axis shows the stressed vowel of the stem form.

Overall, there is some predictability between the stressed vowel of the suffixed form and the stressed vowel of the isolation stem forms. However, these trends appear to be relatively weak compared to the patterns observed for post-tonic reduction.

We can get a general measure of the 'predictability' of the stem stressed vowel from the suffixed form's stressed vowel, by selecting the majority variant for each column in Fig. 2. For example, looking at the rightmost column of Fig. 2, the stressed vowel of the stem is most likely to be [a] if the stressed vowel of the suffixed form is also [a]. This is true for 50 out of 125 (50+18+22+33+2) verbs where the suffixed form's stressed vowel is [a]. In other words, given a suffixed form [puˈtas-an], a speaker could pick the majority variant, and infer that the isolation stem form is [ˈpatas]. This choice predicts the correct output 40% (50/125) of the time. In the third column, we see that if the suffixed form's stressed vowel is [i], the stem form's stressed vowel is most likely to be [e]. In other words, for suffixed forms like [puˈtisan], the stem form is most likely to be [ˈpetis]. Applying this principle, we can correctly predict the stem stressed vowel for 39% (24/62) of relevant forms.

Table 7 summarizes the proportion of forms predicted using this method. Predictability from some vowels (/i/, /a/, and /u/) is fairly low. As a whole, picking the 'best' option based on statistical tendencies in the data only predicts the correct vowel 49% of the time.

| *Suff V* | *Predicted* | *Total* | *% correct* |
|---|---|---|---|
| /o/ | 4 | 4 | 100% |
| /e/ | 54 | 64 | 84% |
| /u/ | 33 | 78 | 42% |
| /i/ | 26 | 68 | 38% |
| /a/ | 50 | 125 | 40% |
| **Overall** | **167** | **337** | **49%** |

Table 7: Predictability of stem stressed vowels from suffixed forms in disyllabic verbs

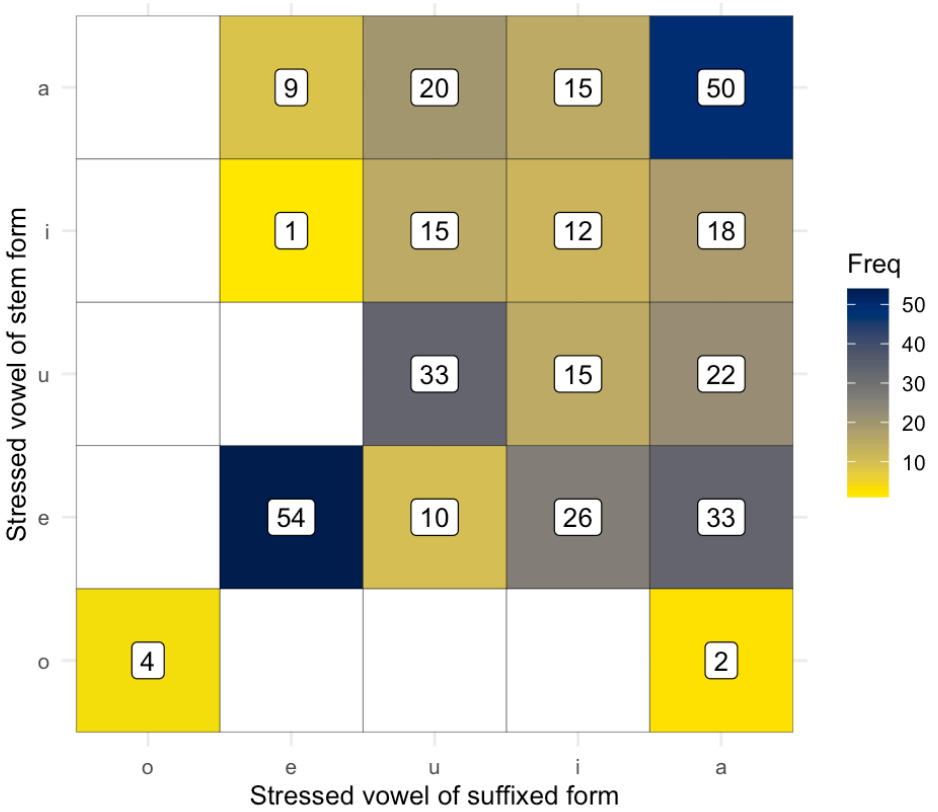


Figure 2: How the pretonic [u] of suffixed forms is realized when stressed in the isolation stem

Because of the low predictability of stem vowels from the suffixed allomorph, the grammar learned by the MGL is not accurate. Table 8 shows the five highest-confidence rules learned by the suffix-base grammar. The grammar learns a general rule that V1 should be restored as [e], but this rule has only a confidence of 0.34. R2 has a high confidence (0.79), but all the other rules are very low in confidence.

| | **rule** | **scope** | **hits** | **reliability** | **confidence** | **example** |
|---|---|---|---|---|---|---|
| R1 | V1=[e] | 341 | 123 | 0.36 | 0.34 | puˈtikan→ˈpetik |
| R2 | If V2=[e]→V1=[e] | 43 | 36 | 0.84 | 0.79 | nuˈtenan→ˈnetun |
| R3 | V2=[a]→V1=[a] | 125 | 50 | 0.40 | 0.37 | duˈyaŋan→ˈdayaŋ |
| R4 | V2=[u]→V1=[u] | 80 | 33 | 0.41 | 0.38 | duˈŋusan→ˈduŋus |
| R5 | V2=[o]→V1=[o] | 4 | 4 | 1.00 | 0.57 | puˈtoŋan→ˈpotuŋ |

Table 8: Examples of rules learned in the vowel-tier suffix-base grammar

### 3.6 Interim summary

Comparison of the stem and suffixed forms revealed a large difference in the relative informativeness of the two paradigm slots. Specifically, the stem forms can be used to predict the suffixed forms with much higher accuracy than the other way around. This asymmetry does not have a purely phonological explanation; it isn’t the case that the stem form is more informative than the suffixed form only because it undergoes fewer phonological neutralization processes. Instead, the informativeness of the stem form is in part due to the very skewed rates of alternation in neutralized segments.

This asymmetry is not predicted by a composite UR analysis; as discussed in Section 2.4, under the composite UR approach, reanalyses of verb paradigms can be based on all cells of the paradigm. As such, this approach makes no predictions about asymmetries between the stem and suffixed forms of Seediq verb paradigms. In contrast, under the single surface base approach, such an asymmetry is expected.

Specifically, it is possible that an older system of Seediq had a more symmetrical distribution of segments. However, as discussed in the beginning of this section, there could have been a gradual restructuring of paradigms, whereby generations of Seediq-learning children have replaced suffixed forms with new forms that obey the pattern of predictability given under the single-surface base hypothesis. The result is the new system observed in the current study, where distributions of segments are strikingly asymmetrical.

For example, statistical patterns in the modern Seediq lexicon reflect a strong dispreference for [ŋ]-[m] alternation. Historically, this dispreference may have been present as a weak statistical tendency (i.e. [m] was already less frequent than [ŋ]). If Seediq speakers have designated the stem to be the base, paradigms which showed the dispreferred [ŋ]-[m] alternation would gradually have been restructured, resulting in the skewed rates of alternation that we see today. Although there is limited historical comparative data available for Seediq, I have elicited one example which suggests this type of reanalysis. As seen in (19), the verb ‘to burn’ is historically [m]-final (Li, 1981; Greenhill et al., 2008), and is therefore expected to show the [ŋ]-[m]alternation. Instead, the suffixed form surfaces with a non-alternating [ŋ].

(19) ˈlauŋ∼luˈuŋan (<*l-um-aum) ‘to burn’
(Li, 1981; Greenhill et al., 2008)

Notably, we cannot rule out the possible that current asymmetries in the lexicon are an artifact of historical sound distributions. In particular, the post-tonic u-e alternation in Tgdaya Seediq results from a sound change of Proto-Austronesian (PAn) *ə to [u] in the final syllable, and to [e] in other environments (Li, 1981). More concretely, PAn stem that is historically *CəCəC (with two schwas) should correspond to a stem-suffix pair like [ˈpetus puˈtesan]. On the other hand, a stem *CəCuC should correspond to a stem-suffix pair like [ˈpetus puˈtusan, with a non-alternating final vowel. Modern Seediq’s tendency towards vowel matching alternation could be an artifact of historical distributions, if historically Seediq had much more CəCəC forms (than CəCuC/CəCiC/CəCaC forms).

Unfortunately, there is almost no direct evidence for historical re-analysis. PAn schwa has reduced to *u in the final position of *all* languages in proto-Atayalic, which encompasses both Seediq and Atayalic (Li, 1981). Additionally, there are very few Seediq stems with established PAn protoforms. This makes it virtually impossible to systematically examine the degree to which reanalysis of post-tonic vowels has been driven by vowel matching.

Despite the lack of direct historical evidence for base-driven reanalysis, there is clear indirect evidence, in terms of the relative informativeness of stem and suffixed forms. In the following section, I present additional evidence that the distribution of vowels in Seediq are skewed in exactly the places expected under reanalysis from the isolation stem. In §5, I present the results of a productivity test, which suggest that regardless of its exact historical origins, in current Seediq, speakers productively extend generalizations from isolation stems to novel suffixed forms.

# 4 An alternative composite UR analysis

So far, I have argued that the asymmetry in informativeness of stem and suffix bases supports the single surface-base approach to morphophonology. In this section, I briefly discuss the alternative composite UR approach, and argue that it does not adequately account for distributions of the Seediq data.

My discussion will focus on vowel alternations in disyllabic stems, because there is vowel neutralization in both stem and suffixed forms, making it possible to compare expected reanalyses from both directions. The case for final consonant alternations is less clear; final consonants are perfectly predictable given a suffixed form, and therefore all reanalyses will be from the stem form.

The composite UR approach differs primarily from the surface-base approach in that reanalyses from both the stem and suffixed forms are possible. In this section, I consider the reanalyses predicted under the two approaches are expected to shape vowel distributions in URs.

There are many possibilities for how learners construct URs when faced with uncertainty in allomorph selection. I will discuss one approach, which is that when a language learner is faced with unpredictability, they guess the UR on the basis of relevant lexical frequencies (Jun & Albright, 2017; Ernestus & Baayen, 2003). For instance, when a Seediq speaker hears [ˈgeru ], they will posit its UR to be /gere / because Seediq stems with underlying /e/ in V1 position are most likely to have underlying /e/ in V2 position. From the UR /gereŋ/, speakers can then infer the suffixed form [guˈreŋan].

Reanalyses should only arise in alternating environments. This means that reanalyses from isolation stems, on the basis of lexical frequencies, should *only* affect URs that surface with a post-tonic [u] in the stem (/CVCuC/, /CVCeC/, /CVCoC/). These reanalyses will specifically result in a preference for (i) /CeCeC/ relative to /CeCoC/ and /CeCuC/, (ii) /CoCoC/ relative to /CoCuC/ and /CoCeC/, and (iii) /CuCuC/ relative to /CuCeC/ and /CuCoC/. Crucially, URs such as /CeCaC/, which go against the vowel matching principle, should nevertheless be preserved because the corresponding stem form [ˈCeCaC] has a non-neutralized post-tonic vowel that is not vulnerable to reanalysis.

Put another way, renalaysis from the stem would result in overrepresentation of URs with /e-e/, /o-o/, and /u-u/ (where /e-e/ is shorthand for /CeCeC/ stems, and likewise for the other vowel-vowel sequences). Vowel sequences such as /e-u/, /u-e/ will be underrepresented in URs, as these are the ones that are vulnerable to reanalysis. However, sequences such as /e-a/ and /e-i/ would not be underrepresented, but are instead expected to occur roughly at chance level. Finally, sequences like /i-i/ and /a-a/, which obey vowel matching but aren't expected as the output of reanalysis, should also occur roughly at chance level.

On the other hand, reanalyses from the suffixed form affects all stem-suffix pairs, as V1 is always neutralized in the suffixed form. Reanalysis from the suffixed form on the basis of lexical frequencies should therefore affect more vowel contexts. Table 9 shows the most frequent V1 given a specific V2; in nearly all vowel contexts, reanalyses the suffixed forms are expected to increase vowel matching. The only exception is that when V2 is /i/, V1 is more likely to be /e/. In general, a frequency-based reanalysis of URs from suffixed forms should result in an increase of the vowel sequences /a-a/, /e-e/, /e-i/, /o-o/, /u-u/ in URs. Additionally, sequences such as /a-u/ and /i-u/, which go against these statistical tendencies, are expected to be underrepresented.

Under the composite UR approach, where reanalyses from both stem and suffixed forms are possible, reanalyses should collectively result in vowel matching across most vowel contexts. In contrast, the surface-base approach predicts that reanalysis will happen from only the unsuffixed stem, and therefore increase

| V2 | V1 | % | UR vowels |
|---|---|---|---|
| a | a | 0.38 | /a-a/ |
| e | e | 0.83 | /e-e/ |
| i | e | 0.37 | /i-e/ |
| o | o | 0.75 | /o-o/ |
| u | u | 0.42 | /u-u/ |

Table 9: Most frequent V1 given target V2; the third column gives the proportion of forms with the target V2 that have the predicted V1; column 4 gives the corresponding UR

vowel matching in more limited contexts. A subset of the vowel sequences predicted to be overrepresented or underrepresented under each approach are summarized in Table 10.

| /V1-V2/ | COMP. UR | SURFACE-BASE |
|---|---|---|
| a-a | + | • |
| e-e | + | + |
| i-i | • | • |
| o-o | + | + |
| u-u | + | + |
| e-u | − | − |
| e-i | + | • |
| e-a | − | • |
| u-e | − | − |
| u-i | − | • |
| u-a | − | • |

Table 10: Predicted UR vowel distributions in composite UR vs. surface-base approaches ('+'=overrepresented, '−'=underrepresented, '•'=at chance)

In order to determine whether certain vowel combinations are significantly more or less common than chance, I use a Monte Carlo procedure (Kessler, 2001; Martin, 2011) to approximate the distribution of the expected rate. Specifically, V1 and V2 of composite URs (constructed from the corpus of 341 stem-suffix pairs) were randomly recombined (vowel position is fixed, so that V1 is always sampled from initial vowels, and V2 from final vowels). After each such shuffling, the number of the target V1V2 pair under the new permutation is recorded. This process was repeated 10,000 times, to give a reliable estimate of the expected (chance value) times the target V1V2 pair should occur.

Fig. 3 shows, for each V1V2 sequence, the 95% confidence interval for chance (found using the Monte Carlo procedure), plotted against the actual number of occurrences in the lexicon. If the actual value is smaller than the 95% confidence interval, then the target V1V2 sequence is underrepresented. Conversely, if the actual value is greater, than it is overrepresented. These values are also summarized and compared against the predictions of different UR structures in Table 11.

In almost all cases, the results of the Monte Carlo simulation are exactly in line with the predictions of the surface-base approach. For example, /e-e/ stems are strongly overrepresented, and /e-u/ stems are underrepresented. In contrast, /e-i/ and /e-a/ sequences both occur at chance levels. This is the distribution that is expected to arise under reanalysis from the isolation stem, where [ˈCeCuC] stems are reanalyzed, while

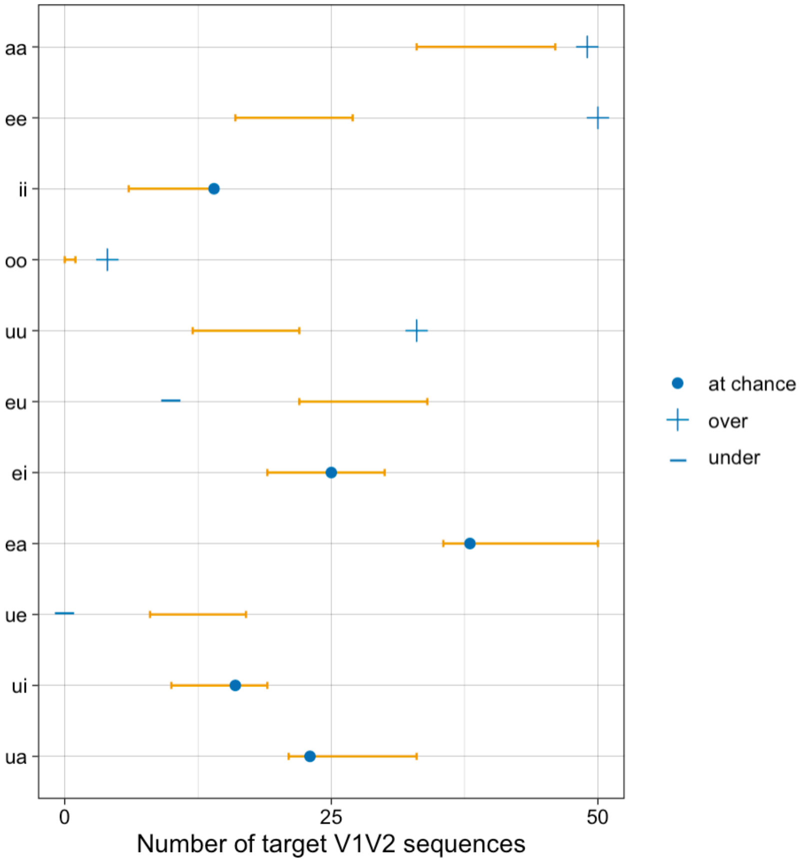


Figure 3: Comparing attested V1V2 pairs to Monte Carlo results.

[ˈCeCaC] and [ˈCeCiC] stems are not. The COMPOSITE UR approach, on the other hand, would predict that sequences such as /e-a/ are also underrepresented, as they would be dispreferred under reanalysis from the suffixed form.

The only place where the results deviate from the predictions of the surface-base approach are the /a-a/ sequences, which are slightly overrepresented in the lexicon. In this case, the number of attested /a-a/ sequences is still relatively close to the distribution at chance level.

In summary, vowel-vowel sequences are over- and underrepresented specifically in environments that would arise under reanalysis from the **surface stem**, but not from the suffixed form. Under a composite UR approach, these asymmetries would be arbitrary. On the other hand, these results provide indirect evidence that reanalyses are overwhelmingly from the stem, rather than from the suffixed form. This directionality falls out naturally from the surface-approach, if the isolation stem is the designated base.

## 5 Productivity of base-driven alternations

The surface-base hypothesis predicts that when given novel stems, speakers should be able to apply alternations in a way that makes suffixed forms more predictable from stems. The stem-base grammar learned by the MGL in §3 makes more specific predictions; in particular, it predicts that [u]∼[e/o] alternations will be extended in stems with a stressed [e/o].

| /V1-V2/ | comp. UR | surface-base | Result |
|---|---|---|---|
| a-a | + | ● | + |
| e-e | + | + | + |
| i-i | ● | ● | ● |
| o-o | + | + | + |
| u-u | + | + | + |
| e-u | − | − | − |
| e-i | + | ● | ● |
| e-a | − | ● | ● |
| u-e | − | − | − |
| u-i | − | ● | ● |
| u-a | − | ● | ● |

Table 11: Predicted UR vowel distributions in against Monte Carlo results ('+'=overrepresented, '−'=underrepresented, '●'=at chance)

This section discusses the results of a production experiment testing predictions for post-tonic vowel alternation. Results suggest that speakers productively apply vowel matching alternations to post-tonic [u] but not other vowels, in line with the predictions of the surface-base approach. However, speakers have also learned vowel matching non-veridically, extending it to environments not predicted by the model developed in §3.

The experimental methodology adopted was a modified version of a nonce-word task (i.e. wug test; Berko, 1958). Production experiments following this paradigm have been shown to elicit responses that, when averaged over several speakers, replicate distributional facts about the lexicon (e.g. Zuraw, 2000; Ernestus & Baayen, 2003, and many others). However, instead of nonce words, the current study uses 'gapped forms', or stems with no known suffixed forms. This was done out of respect for my participants, who raised concerns that the use of nonce words in experiments would interfere with ongoing language revitalization efforts.

In Section 5.1 below, I summarize the predictions and results of the experiment. Detailed description of the experiment procedure can be found in Kuo (2020, 2022).

## 5.1 Predictions

Stimuli consisted of disyllabic stems ending in closed syllables (i.e. CVCVC), where the first vowel (V1) was one of /a, e, u/ and the second vowel (V2) was one of /a, u/. This results in six possible vowel combinations, summarized in Table 12. These vowel combinations were selected to elicit a range of environments in which post-tonic [u] is expected to either alternate with [e] or not alternate. Stems with a post-tonic /a/ are expected to never show V2 alternations.

Stimuli where V1 was /i,o/ were not included for several reasons. First, I limited the conditions tested in order to keep the experiment under two hours (the experiment currently runs around 90 minutes per participant). In addition, there were also relatively few words where V1 was [i] or [o], and almost all of these were found to be non-suffixable during a pilot study.

Predicted responses, on the basis of the rule-based model in §3, are summarized in Table 12; the rightmost column gives an example stimulus for each condition, with the expected preferred outcome given in parentheses. If speakers generalize the vowel matching pattern, they should apply the [u]∼[e] alternation to most CeCuC stimuli. In CaCuC stems, non-alternation should be preferred, while [u]∼[e] alternation is present as a minority pattern. In CuCuC stems, vowel alternation should never be observed, since the faithful non-alternating outcome already satisfies vowel matching.

| V1 | V2 | Prediction | Example |
|---|---|---|---|
| a | u | disprefer alternation | ˈdaruk (duˈruk-an) ‘oil’ |
| e | u | [u]∼[e] alternation | ˈkeruŋ (kuˈreŋ-an) ‘wrinkles’ |
| u | u | never alternate | ˈcuguk (cuˈguk-an) ‘Bidens plant’ |
| a | a | V2 never alternates | ˈsabak (suˈbak-an) ‘dregs, pulp’ |
| e | a |  | ˈrehak (ruˈhak-an) ‘seed’ |
| u | a |  | kuˈsuwak (kusuˈwak-an) ‘yawn’ |

Table 12: Experimental conditions: vowel alternations

## 5.2 Results

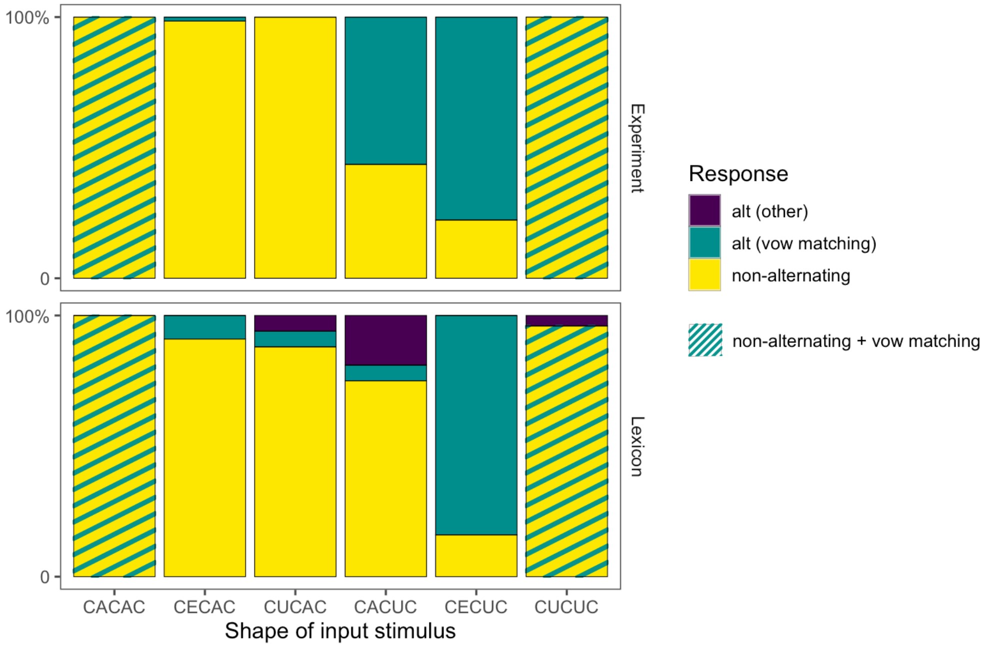


Figure 4: Vowel alternation rates in experiment vs. lexicon

Results for final vowel alternations are given in Fig. 4, which shows the proportion of response types by vowel condition. The distribution of vowels in the lexicon is given on the bottom column for reference. Cases where vowel alternation obeyed the vowel matching pattern (i.e. resulted in the stem and suffixed forms having the same stressed vowel) are indicated in green.

First, looking at the left-hand column, which shows results for stems with a post-tonic [a], we see that consistent with predictions, final /a/ almost never alternates. There was one exception ([huˈrenaŋ]∼[huruˈneŋi]); in this case, an [a∼e] alternation resulted in vowel matching. On the right-hand column, in line with predictions, CeCuC stems prefer the [u]∼[e] alternation, and CuCuC stems never alternate.

However, speakers deviated from the lexicon in stems of the form CaCuC. [u]∼[e] alternation was not observed at all. Instead, for around half of the stems in this category, speakers applied a novel [u∼a] alternation (e.g. [ˈdar**u**k]∼[duˈr**a**k-an]. This alternation is irregular and novel, in the sense that it is predicted by neither lexical statistics nor the stem-base grammar. Instead, it appears that speakers have extended the vowel matching pattern to CaCuC stems.

### 5.3 Discussion and interim summary

Overall, experimental results provide indirect support for the surface-base hypothesis, and suggest that speakers have productively learned to apply vowel alternations in a way that renders suffixed forms more predictable from stem forms. Specifically, speakers learned a vowel matching pattern, but appear to have generalized it beyond CeCuC and CoCuC stems, resulting in [u∼a] alternations for CaCuC stems.

The fact that speakers did not just match lexical statistics, but instead overgeneralized vowel-matching to CaCuC forms, supports the conclusion that speakers have learned a productive process for predicting vowel alternations using a stem base. Notably, however, over-extension of vowel matching cannot be accounted for in grammars of morphological mappings formed purely on the basis of lexical distributions (as was put forth in §3). Instead, speakers have generalized patterns that are rooted in phonological principles (Hayes et al., 2009; Becker et al., 2011; Moore-Cantwell, 2013). In fact, I will argue that instead of learning arbitrary stem→suffix mappings, speakers have generalized two phonologically-grounded principles: a tendency for stressed vowels of morphologically-related forms to match (prosodic correspondence; Crosswhite, 1998), and a preference for sonorous vowels in stressed positions (prominence alignment; Kenstowicz, 1994). In the following section, I propose a surface-base analysis of Seediq, and adopt a constraint-based approach that captures these two tendencies.

Additionally, I propose that speakers' overextension of vowel matching arises from a complexity bias, where they learned a simpler constraint which enforces vowel matching on all vowels, instead of a narrowly-defined constraint which affects only mid vowels. Note that my analysis implicitly assumes that speakers have learned a vowel matching pattern across all vowels. The stimuli doesn't contain stems with stressed [i], so potential follow-up work could confirm whether vowel matching holds for CiCuC stems. The effect of a learning bias is explored in Section 6.3,

## 6 A constraint-based model of Seediq vowel alternations

§3 assessed the stem-suffix asymmetry in Seediq using a model which learns morphological correspondences from lexical distributions. Experimental results suggest that this model does not provide a sufficient *theoretical* account of Seediq verbal alternations. In particular, speakers applied a novel [u]∼[a] alternation to CaCuC forms. I argue that this is because, instead of directly learning morphological mappings of [u]→[e,o], speakers have learned a more general vowel matching principle.

In this section, I aim to unify lexical and experimental results, by proposing a stem-base model of Seediq vowel alternations, which uses a phonological constraint on vowel matching to motivate alternation. The analysis will be set in the framework of Maximum Entropy Harmonic Grammar (MaxEnt; Goldwater & Johnson, 2003), a stochastic variant of Optimality Theory (Smolensky, 1986; Prince & Smolensky, 1993).

MaxEnt is a probabilistic variant of Harmonic Grammar (Legendre et al., 1990; Pater, 2009), which are themselves variants of OT that use weighted (instead of ranked) constraints. MaxEnt generates a probability distribution over the set of candidate outputs based on their violations of a set of weighted constraints.

Unlike classic OT, where strict ranking ensures that losing candidates never surface, all candidates in MaxEnt grammars receive some probability. However, if constraint weights are sufficiently different, MaxEnt produces results that are functionally very similar to classic OT, where the winning candidate gets near-perfect probability, while losing candidates get near-zero probability.

MaxEnt models are associated with learning algorithms that have been proved to converge on one optimal solution (Berger et al., 1996), which has the maximum log-likelihood. To learn model constraint weights, I use Excel's Solver add-in (Generalized Reduced-Gradient Algorithm; Fylstra et al., 1998).

Due to space constraints, this section and subsequent modeling will discuss only vowel alternations, where effects of stem-based reanalysis are backed up both lexical statistics and experimental evidence. An in-depth MaxEnt analysis of consonant alternation can be found in Kuo (2020).

The rest of this section is organized as follows. Section 6.1 will introduce a base-driven analysis of Seediq vowel alternations, fit to the lexicon. In Section 6.3, I propose to model speakers' overextension of vowel matching in the experiment as the result of a bias towards simpler, less complex grammars. Specifically, I compare two models, one where complexity bias is modeled using a smoothing term (Martin, 2011), and one which explicitly penalizes more complex constraints by penalizing them for having high weight (White, 2013, 2017). Both models significantly outperform the baseline, unbiased grammar.

## 6.1 A base-driven model of vowel alternations in the Seediq lexicon

This section introduces the constraints needed under a base-driven approach to Seediq post-tonic vowel alternations. The following section (6.2) will then present weights learned algorithmically in models trained on the lexicon.

### 6.1.1 Inputs and faithfulness

Following Albright (2002a,b, 2010, et seq.), I assume that the input to the model is the base form, in this case the isolation stem. Essentially, inputs are segmentally identical to isolation stems, but are under-specified for stress (as stress is non-phonemic). For example, given a stem-suffix pair like [ˈpukuc], the input to the model is /pukuc/. When deriving the suffixed form, candidates are in both an input-output (IO) correspondence relationship with these URs (e.g. /pukuc/), and an output-output (OO) correspondence relationship with the surface unsuffixed stems (e.g. [ˈpukuc]).

Additionally, inputs were schematized into CVCVC forms with all possible surface vowel combinations (i.e. the first vowel V1 was one of [a, e, i, o, u], while V2 was one of [i, a, u]). Vowel-final forms (e.g.[ˈqene]) were omitted. Since all pre-tonic vowel neutralization processes are exceptionless, I ignored the difference between i) stems with an initial onsetless syllable (e.g. [ˈatak]), ii) CVHVC stems (where H is [h] or [ʔ]), and iii) CVCVC stems. Idealized training data was used for ease of interpretation, and so that results could be compared with experimental data.

The effects of IO-Faithfulness and OO-Faithfulness are demonstrated in tableau (20), which shows how suffixed forms are derived for the hypothetical input /petus/.[3] The constraints enforcing vowel alternation will be introduced in Section 6.1.3; for now, it is written as MatchV. Candidate (b), which undergoes vowel alternation to satisfy MatchV, violates faithfulness constraints, written in the tableau as IO-Faith and OO-Faith. However, because both faithfulness constraints have lower weight than the constraint which enforces vowel matching, candidate (b) is still preferred.

(20) *IO-faithfulness*

|  |  |  | *OO-MatchV* | *IO-Faith* | *OO-Faith* |
|---|---|---|---|---|---|
| /petus-an/ ∼[ˈpetus] | P | $\mathcal{H}$ | 7 | 2.5 | 2.5 |
| a. puˈtus-an | 0.12 | 7 | 1 |  |  |
| b. puˈtes-an | 0.88 | 5 |  | 1 | 1 |

In this tableau, IO and OO-Faithfulness constraints have the same violation profile. In general, because all IO and OO-Faithfulness constraints have the exact same violation profile, I will omit IO-Faithfulnes from future tableau.

### 6.1.2 Constraints for pre-tonic vowel neutralizations

Pretonically, vowels either delete, assimilate to an stressed vowel, or reduce to [u]. All three patterns can be motivated by fairly standard markedness constraints. In this section, I discuss only pretonic vowel reduction; analyses for vowel assimilation and deletion are found in Kuo (2020).

Pretonic vowel reduction to [u] can be motivated by prominence alignment, where vowels reducing to less sonorous variants in non-prominent positions (Kenstowicz, 1994; Crosswhite, 2000). However, a sonority-driven reduction account does not explain why /i/ and /e/ reduce to [u]. In fact, Seediq vowel reduction to [u] is saltatory (White, 2013; Hayes & White, 2015), because /e/ reduces to [u] instead of the phonetically closer (and equally unmarked) [i]. Barnes (2002) speculates that the synchronic pretonic reduction pattern originated from a transparent process of pretonic vowels merged to a central high vowel. The pattern was obscured by subsequent phonetic rounding and backing of the reduced vowel to [u].

Saltatory alternations are problematic in parallel OT (Łubowicz, 2002; Ito & Mester, 2003; Hayes & White, 2015); even though pretonic neutralization to [u] is consistent with a prominence alignment account, it cannot be straightforwardly analyzed using standard markedness and faithfulness constraints. As such, I adopt a parsimonious constraint that describes the reduction pattern. This constraint, License[u]/pretonic (Lic[u]/pret), is defined in (21), and essentially penalizes non-[u] syllables in pretonic position. This is demonstrated in tableau (22) for [ˈbarah]∼[buˈrahan].

The faithful candidate (a) fatally violates the highly weighted Lic[u]/pret, and is assigned near-zero probability. Candidate (b), which repairs the Lic[u]/pret violation by incurring a violation of OO-Ident[high] (and other identity constraints), has the highest probability. Candidate (c), which repairs the markedness

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[3] Although it is not discussed here, both candidates (a) and (b) undergo pretonic reduction.

violation by deleting the stem's initial syllable, is ruled out due to violation of higher-weighed faithfulness constraints.

(21) License[u]/*pretonic*: non-[u] vowels cannot appear in pretonic syllables.

(22) *Pretonic vowel reduction.* Very small probabilities (on the order of $10^{-5}$) are listed as zero.

| | | | *Lic[u]/pret* | Max-V | Max-C | *OO-ID[high]* |
|---|---|---|---|---|---|---|
| /barah-an/∼[ˈbarah] | P | $\mathcal{H}$ | 20 | 10 | 10 | 1 |
| a. baˈrah-an | 0 | 20 | 1 | | | |
| b. buˈrah-an | 1 | 1 | | | | 1 |
| c. ˈrah-an | 0 | 20 | | 1 | 1 | |

### 6.1.3 Constraints for post-tonic vowel alternations

Post-tonically, [u] alternates with [e, o] to satisfy vowel matching; alternation is preferred only when it results in the stressed vowel of the suffixed form matching the stressed vowel of the stem.

I propose that speakers have learned both a specific constraint enforcing vowel matching in mid vowels, and a more general constraint enforcing vowel matching across all vowel categories. The choice to include the more general constraint is motivated by the experimental results, where speakers extended the vowel matching pattern, applying an innovative [u]∼[a] alternation to CaCuC stems (Section 5.2).

Both the specific and general constraints are formalized using prosodic correspondence (Crosswhite, 1998).[4] These are constraints which enforce identity between prosodic elements of related output forms, rather than linearly related segmental units. In particular, I use the two constraints Nuc-Ident-OO(Mid-V) and Nuc-Ident-OO(V), to enforce vowel matching. These constraints, defined in (23), set up a correspondence relation between the stressed syllable nuclei of related output forms, and requires that these positions have the same vowel.

(23) a. Nuc-Ident-OO(MidV): For $\alpha$, a stressed **mid-vowel** nucleus of the base, and $\beta$, a stressed nucleus of an output, where $\alpha$ corresponds to $\beta$, $\alpha$ and $\beta$ must be the same.

b. Nuc-Ident-OO(V): For $\alpha$, a stressed nucleus of the base, and $\beta$, a stressed nucleus of an output, where $\alpha$ corresponds to $\beta$, $\alpha$ and $\beta$ must be the same.

Ident constraints typically reference feature specifications. For Seediq, the constraints enforce total identity. This approach builds on evidence that total identity is distinct from partial identity, and that constraints which enforce total identity are necessary (Coetzee & Pater, 2008; Gallagher & Coon, 2009).

Tableau (24) demonstrates how the two vowel matching constraints are used to derive outputs that match lexical frequencies.[5] The model learns a much higher weight for Nuc-ID(MidV) than for Nuc-ID(V), reflecting how in the lexicon, vowel-matching alternation only happens for CeCuC and CoCuC stems. For the

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[4] In Kuo (2022), I go into a more detailed discussion of why Seediq vowel matching should be analyzed as prosodic correspondence.
[5] The constraint *P/i,u will not be introduced until the following section, but is not crucial to this tableau.

input /putus/, the faithful candidate satisfies Nuc-ID(V), and receives high probability. For /petus/, the high weight of Nuc-ID(MidV) relative to competing faithfulness constraints causes candidate (e), which undergoes [u]∼[e] alternation, to be preferred over the faithful candidate. Candidate (f) is ruled out because vowel alternation does not resolve violations of Nuc-ID(MidV). For /patus/, candidate (j), which undergoes [u]∼[a] alternation to resolve Nuc-ID(V) violations, is ruled out because the weight of Nuc-ID(V) is much lower than that of competing faithfulness constraints. As will be addressed in **??**, this differs from the experiment, where speakers did in fact apply the [u]∼[a] alternation to around 50% of CaCuC forms.

(24) *Tableau: post-tonic vowel alternations for CVCuC inputs.* The probability of each candidate in the lexicon (Obs.) is shown alongside model predictions (P).

| | | Obs | P | $\mathcal{H}$ | *Nuc-ID(MidV)* | *Nuc-ID(V)* | **P/i,u* | *ID[low]* | *ID[high]* | *ID[front]* |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 5.18 | 0.66 | 3.89 | 0.99 | 5.98 | 2.20 |
| /putus-an/∼[ˈputus] | | | | | | | | | | |
| a. | puˈtusan | 1 | 0.93 | 3.89 | | | 1 | | | |
| b. | puˈtesan | 0 | 0.01 | 8.84 | | 1 | | | 1 | 1 |
| c. | puˈtosan | 0 | 0.06 | 6.65 | | 1 | 1 | | 1 | |
| /petus-an/∼[ˈpetus] | | | | | | | | | | |
| d. | puˈtusan | 0.20 | 0.18 | 9.68 | 1 | 1 | 1 | | | |
| e. | puˈtesan | 0.80 | 0.81 | 8.15 | | | | | 1 | 1 |
| f. | puˈtosan | 0 | 0.02 | 12.10 | 1 | 1 | | | 1 | |
| /potus/∼[ˈpotus] | | | | | | | | | | |
| g. | puˈtusan | 0 | 0.04 | 9.68 | 1 | 1 | 1 | | | |
| h. | puˈtosan | 1 | 0.96 | 6.40 | | | | | 1 | |
| i. | puˈtesan | 0 | 0 | 13.84 | 1 | 1 | | | 1 | 1 |
| /patus/∼[ˈpatus] | | | | | | | | | | |
| j. | puˈtusan | 0.76 | 0.90 | 4.09 | | 1 | 1 | | | |
| k. | puˈtesan | 0.18 | 0.01 | 8.26 | | 1 | | | 1 | 1 |
| l. | puˈtosan | 0 | 0.08 | 6.51 | | 1 | | | 1 | |
| m. | puˈtasan | 0.06 | 0 | 9.42 | | | | 1 | 1 | 1 |

### 6.1.4 Saltation in post-tonic vowel alternations

In tableau (24) above, the model generally over-predicts rates of [u]∼[e] alternation, while over-predicting rates of [u]∼[o] alternation. For the input /putus/, the model assigns the [u]∼[o] alternating candidate (c) a probability of 0.06, whereas in the lexicon this candidate is never observed. Similarly, for /patus/, the model assigns too much probability (P=0.08) to [puˈtosan], and too little probability to [puˈtesan] (P=0.01).

This issue arises because, similar to pretonic vowel reduction, post-tonic vowel alternations in Seediq are saltatory, in that post-tonic [u] prefers to alternate with [e] instead of the phonetically closer [o]. This results in a constraint weighting conflict; [u]∼[e] alternating candidates will always incur more faithfulness constraint violations than [u]∼[o] alternating forms, and therefore (when all else is held equal) be assigned lower probability.

To resolve this conflict, I use a constraint *[o], which simply penalizes surface [o]. Although this constraint is not rooted in a clear markedness motivation, it does capture fact that [o] is marginal in the Seediq lexicon, and occurs at a much lower frequency than the other vowels. An alternative approach to dealing with saltation, which Hayes & White (2015) adopt, is to introduce the *Map family of faithfulness constraints Zuraw (2010, 2013). *Map are less restrictive than classical faithfulness constraints, and can describe the correspondence between any two natural classes of sounds, even when the two classes differ in more than one feature. Using *Map constraints, it is therefore possible to have both a constraint penalizing a change of [u]∼[e], and a constraint penalizing a change of [u]∼[o]. I explore this approach for Seediq in Kuo (2020).

The effect of *[o] is demonstrated for /putus/ and /patus/ in tableau (25), which is identical to tableau (24), other than the addition of *[o]. For both inputs, the [u]∼[o] alternating candidate receives much lower weight because they incur violations of *[o].

(25) *Tableau: Effect of *o on /putus/ and /patus/.*

|  |  |  |  | *Nuc-ID(MidV)* | *Nuc-ID(V)* | **P/i,u* | *o | *ID[low]* | *ID[high]* | *ID[front]* |
|---|---|---|---|---|---|---|---|---|---|---|
|  | Obs | P | $\mathcal{H}$ | 3.80 | 0.69 | 0.49 | 3.86 | 0.67 | 0.32 | 3.04 |
| /putus-an/∼[ˈputus] |  |  |  |  |  |  |  |  |  |  |
| a. puˈtesan | 0 | 0 | 7.85 | 1 | 1 |  |  |  | 1 | 1 |
| b. puˈtusan | 1 | 1 | 0.49 |  |  | 1 |  |  |  |  |
| c. puˈtosan | 0 | 0 | 8.35 | 1 | 1 |  | 1 |  | 1 |  |
| d. puˈtasan | 0 | 0 | 8.52 | 1 | 1 |  |  | 1 | 1 | 1 |
| /patus-an/∼[ˈpatus] |  |  |  |  |  |  |  |  |  |  |
| e. puˈtusan | 0.76 | 0.84 | 1.18 |  | 1 | 1 |  |  |  |  |
| f. puˈtesan | 0.18 | 0.06 | 3.85 |  | 1 |  |  |  | 1 | 1 |
| g. puˈtosan | 0.00 | 0.04 | 4.17 |  | 1 |  | 1 |  | 1 |  |
| h. puˈtasan | 0.06 | 0.06 | 3.83 |  |  |  |  | 1 | 1 | 1 |

### 6.1.5 Explaining asymmetries in vowel alternation

In the experimental results for post-tonic vowel alternation, speakers applied a novel alternation to post-tonic [u], resulting in [u]∼[a] alternations. However, they never applied [a]∼[u] alternations to post-tonic [a], even when doing so would resolve violations of Nuc-ID(V).

There are multiple possible explanations for this; speakers could, for example, have learned a source-oriented generalization that only post-tonic [u] can alternate Becker & Gouskova (2016). However, source-oriented generalizations are difficult to capture in OT using standard markedness constraints, which target the output form. Instead, they require positing additional mechanisms. For example, highly ranked *directional* faithfulness constraints could be introduced to the grammar, to protect post-tonic /a/ and /i/ from alternating with [u] (but would not, for example, prevent /u/ from alternating with [a]) .

As an alternative, I analyze the directionality in vowel alternations as a preference for more sonorous vowels in stressed positions; [u]∼[a] alternations are preferred over [a]∼[u] alternations because the former increases the sonority of the stressed syllable, while the latter does the opposite. This approach captures the generalization that Seediq vowel alternations are prominence-aligning; pretonic vowels raise to the less sonorous [u], while stressed vowels prefer to be more sonorous. Typologically, a preference for sonorous vowels in foot-peaks (i.e. main-stressed syllables) is also well-attested, and has been observed in languages (e.g. Crosswhite & Jun (2001) on Zabiče Slovene and Chung (1983) on Chamorro.

Kenstowicz (1994) formalizes this preference for sonorous vowels in foot-peaks using a family of constraints, where less sonorous vowels are relatively more constrained from appearing in stressed positions. These constraints are given in (27); they penalize certain vowels in word-peaks, and are argued to follow a universal ranking hierarchy. In a weighted constraint model like MaxEnt, this means that a constraint like *P/ə should always have higher weight than *P/e,o. I will adopt this approach, and specifically use the constraint *P/i,u.[6] Other foot-peak constraints such as *P/a are assumed to be in the grammar, but are not active because of their relatively low weight.

(26) *Universal hierarchies and rankings for foot-peaks (Kenstowicz, 1994).*
   a. Hierarchy for foot peaks: a >e,o > i,u >ə
   b. Constraint formulation of*P/*x*: assign a violation to every vowel *x* that is in a foot peak (i.e. the nucleus of a stressed syllable)
   c. Constraint ranking: *P/ə ≫ *P/i,u ≫ *P/e,o ≫ *P/a

The effect of *P/i,u is demontrated in tableau (27), which compares the model's predictions of /patus/ and /putas/. Note that although *P/i,u is needed to explain experimental results, it has a very small effect in a lexicon-trained model. This is because, excluding some irregularities, only [u] alternates post-tonically. Consequently, there is little evidence to disambiguate between alternations which improve or reduce the stressed syllable's sonority.

Nevertheless, the model does learn a slight preference for [u]∼[a] alternation over [a]∼[u] alternation. For the input /patus/, candidate (d), which undergoes [u]∼[a] alternation, receives some probability (P=0.06) because alternation reduces violations of both NUC-ID(V) and *P/i,u. In contrast, for the input /putas/, candidate (a), which undergoes [a]∼[u] alternation to resolve the violation against NUC-ID(V), receives zero probability. The reason is that in this case, alternation actually increases violatons of *P/i,u.

(27) *Effect of *P/i,u* on vowel alternation

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[6] More sophisticated models of gradient constraints on syllable weight are explored in work like Flemming (2001) and Ryan (2011).

| | | | | *Nuc-ID(V)* | **P/i,u* | **[o]* | *ID[low]* | *ID[high]* | *ID[front]* |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 0.90 | 0.98 | 4.04 | 0.48 | 0.00 | 3.56 |
| /patus/∼[ˈpatus] | | | | | | | | | |
| a. puˈtusan | 0.76 | 0.84 | 1.18 | 1 | 1 | | | | |
| b. puˈtesan | 0.18 | 0.06 | 3.85 | 1 | | | | 1 | 1 |
| c. puˈtosan | 0.00 | 0.04 | 4.17 | 1 | | 1 | | 1 | |
| d. puˈtasan | 0.06 | 0.06 | 3.83 | | | | 1 | 1 | 1 |
| /putas/∼[ˈputas] | | | | | | | | | |
| e. puˈtusan | 0 | 0.00 | 5.02 | | 1 | | 1 | 1 | 1 |
| f. puˈtesan | 0 | 0.02 | 4.95 | 1 | | | 1 | | 1 |
| g. puˈtosan | 0 | 0.02 | 4.94 | 1 | | 1 | | 1 | |
| h. puˈtasan | 1 | 0.95 | 0.90 | 1 | | | | | |

Notably, this prosodic alignment predicts that sonority of the alternating vowel will affect how readily speakers extend vowel matching. For one, speakers should disprefer extension of vowel matching when doing so reduces the stressed vowel's sonority. For example, the post-tonic vowel in CiCaC stem could undergo [a]∼[i] alternation to satisfy Nuc-ID(V). However, because alternation of [a] to [i] increases violations of *P/i,u, [a]∼[i] alternation should occur at a lower rate than [u]∼[a] alternation did for CaCuC stems. Predictions like this are hard to test using just corpus data, as post-tonic [i] and [a] generally never alternate in the lexicon. However, they could potentially be tested in future experimental work.

## 6.2 Model fit to lexicon vs. experimental results

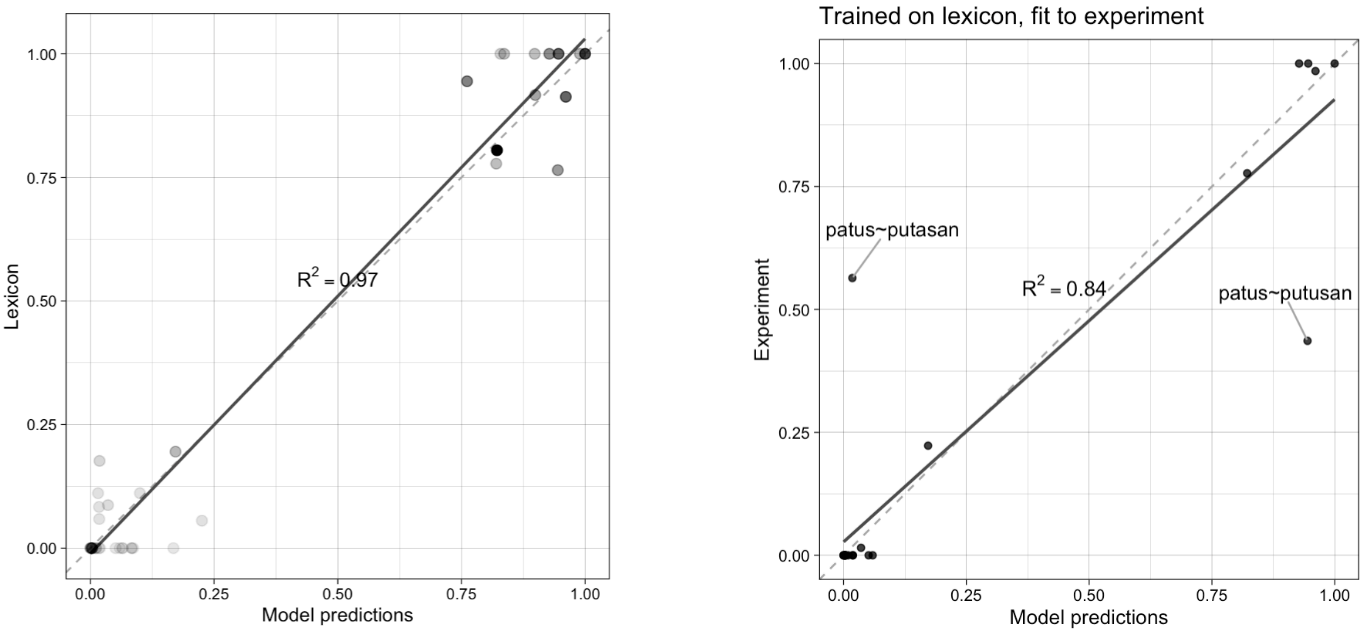
The constraints presented in Section 6.1 were used to train a MaxEnt model. All markedness constraints were confirmed to be significant using likelihood ratio tests (Wasserman, 2004; Hayes et al., 2012). Additionally, all relevant faithfulness constraints were included even if they ended up testing non-significant.

The vowel alternation model has 7 constraints and a log-likelihood of -69.4 (null=-273.6). Fig. 5a compares model predictions against the lexicon; each point represents a candidate's mean predicted probability against its observed probability. In this figure, the model closely fits the lexicon ($r^2 = 0.97$). Additionally, the optimal weights assigned to the training data are shown in (28). From here, we see that the model has learned a much higher weight for Nuc-ID(MidV) relative to Nuc-ID(V); this matches the lexicon, where CeCuC and CoCuC inputs undergo vowel-matching alternations, while other inputs don't.

(28) *Constraint weights*

| | | | |
|---|---|---|---|
| Nuc-ID(MidV) | 5.33 | *[o] | 4.71 |
| Nuc-ID(V) | 0.17 | ID[high] | 2.10 |
| *P/i,u | 0.72 | ID[low] | 0.20 |
| ID[front] | 2.55 | | |

Although the model is able to closely match the lexicon, it does less well in matching the experimental



(a) (b)

Figure 5: Model predictions plotted against lexicon (a, left) vs. experimental results (5, right). Fitted regression lines are included. Darkness of the points corresponds to relative frequency.

results. In the experiment, speakers over-extended the vowel-matching alternation to CaCuC forms, resulting in alternations such as [ˈpatus]∼[puˈtasan]. The model is not able to account for this; when fit to the epxerimental results, it severely under-predicts rates of [u]∼[a] alternation. This is demonstrated in Fig. 5b, which compares the fit of the model to the lexicon and to experimental results. As seen in Fig. 5b, the model mostly achieves a close to fit to experimental results, but makes very wrong predictions for /patus/. In the experiment, speakers produced the [u]∼[a] alternating form around 56% of the time, but the model predicts that this candidate only occurs 6% of the time.

## 6.3 Explaining experimental results as a complexity bias

In the experimental results, speakers successfully learned generalizations which made alternations more predictable from a stem base. At the same time, speakers over-extended vowel matching beyond the environments observed in the lexicon. In this section, I discuss how the model above could be modified to learn over-extension of vowel matching.

In general, non-viridical learning may arise from different types analytic bias (Moreton, 2008). I propose specifically that the Seediq experimental results follow from a complexity bias, where learners preferentially learned the more general constraint Nuc-ID(V), over the more complex constraint Nuc-ID(MidV) (that is specific to only mid vowels). This analysis would be in line with a body of work suggesting that people preferentially learn simpler constraints (Pycha et al., 2003; Moreton & Pater, 2012)

### 6.3.1 Bias implementation

In MaxEnt, bias can be implemented as a Gaussian prior (Wilson, 2006; Martin, 2011; White, 2013). The **bias term**, calculated as in (29), is defined in terms of a mean ($\mu$) and standard deviation ($\sigma$). For each constraint, $w$ is its learned weight, and $\mu$ can be thought of as the 'preferred' weight. The more the learned weight $w$ deviates from $\mu$, the larger the numerator will be. This, in turn, corresponds to a larger penalty resulting from the bias term.

(29) bias term=$\sum_{i=1}^{m}\frac{(w_i-\mu_i)^2}{2\sigma^2}$

The value of $\sigma^2$ determines how much effect the preferred weight ($\mu$) has; lower values of $\sigma^2$ result in a smaller denominator, and therefore greater penalty for weights that deviate from their $\mu$. In a biased model, the objective function being maximized is the log likelihood subtracted by the prior term. In principle, both $\mu$ and $\sigma^2$ can be varied to give constraints a preference towards a certain weight. In the current models, $\sigma^2$ is set to fixed values, and $\mu$ is varied.

I implement two types of grammars with a complexity bias. The first grammar, which I will call the Uniform grammar, simply assigns all constraints a uniform $\mu$ value (Martin, 2011). The bias term essentially acts as a smoothing term, so that the model prefers to assign uniform weight to constraints (therefore penalizing complex grammars). For Seediq, because the grammar has both general and vowel-specific constraints for vowel matching, the smoothing term would cause the general matching constraint to be assigned some weight, resulting in an over-extension of vowel matching.

For the Uniform grammar, $\mu$=2.5 and $\sigma$=0.8. In principle, $\mu$ can be any low value, as long as it is uniform across all the constraints. Here, I follow White (2017) in setting $\mu$ to the average of all constraint weights learned in the baseline (unbiased) grammar. The value of $\sigma$ was set by testing a range of values from 0.5-100, and finding the one that produced the best-fit model.

The second grammar I test, termed the Biased grammar, assigns Nuc-ID(MidV) is assigned a lower $\mu$ of zero. In all other respects, it is identical to the Uniform grammar. This method of implementing bias by varying $\mu$ has been in explored in work such as White (2013); Hayes & White (2015); White (2017). The resulting grammar has one more parameter than the Uniform grammar, but also has a stronger complexity bias in that it directly penalizes Nuc-ID(MidV) for having high weight.

### 6.3.2 Model results and fit

Table 13 compares the weights learned in the Baseline model (which has no bias term) against the Uniform and Biased grammars. As expected, the Uniform model learns more equal weights across-the-board. In particular, it learns a relatively higher weight for both the general vowel-matching constraint, and *P/i,u (which blocks alternations like [ˈputas]∼[puˈtusan]). The Biased model learns an even higher weight for Nuc-ID(V), and a slightly lower weight for Nuc-ID(MidV) compared to the other two models.

Overall, these differences strongly improve model fit to experimental results, as summarized in Table 14. First, comparing the Baseline and Uniform modles, adding a smoothing term significantly increases log-likelihood of the model fit to experimental results (p = 8 x $10^{-21}$, df = 2). Compared to this model, the Biased model also performs significantly better (p=5.6 x $10^{-}$8, df=1).

|  | baseline | Uniform | Biased |
|---|---|---|---|
| Nuc-ID(MidV) | 5.33 | 4.49 | 3.25 |
| Nuc-ID(V) | 0.17 | 1.85 | 2.24 |
| *P/i,u | 0.72 | 1.97 | 2.17 |
| *[o] | 4.71 | 3.92 | 3.56 |
| ID[low] | 0.20 | 0.33 | 0.58 |
| ID[high] | 2.10 | 3.64 | 3.24 |
| ID[front] | 2.55 | 2.52 | 2.12 |

Table 13: Constraint weights learned in baseline vs. biased models

|  | **R$^2$** | $\hat{\mathbf{L}}$ |
|---|---|---|
| Baseline | 0.84 | -411.0 |
| Uniform | 0.87 | -306.6 |
| Biased | 0.91 | -266.0 |

Table 14: Proportion of variance explained (R$^2$) and log-likelihood ($\hat{L}$) of model predictions fit to experimental results

The improvement in model fit is driven by the model's predictions for /patus/. Fig. 6 plots the predictions of the Baseline, Uniform, and Biased models against the experimental results. Note that these plots only show the vowel conditions that were tested in the experiment. As already discussed, the Baseline model under-predicts the rate of [u]∼[a] alternation for /patus/ (P=0.02). In contrast, both the Uniform and Biased models correctly predict a higher rate of [u]∼[a] alternation, with the Biased model performing better (P=0.17).

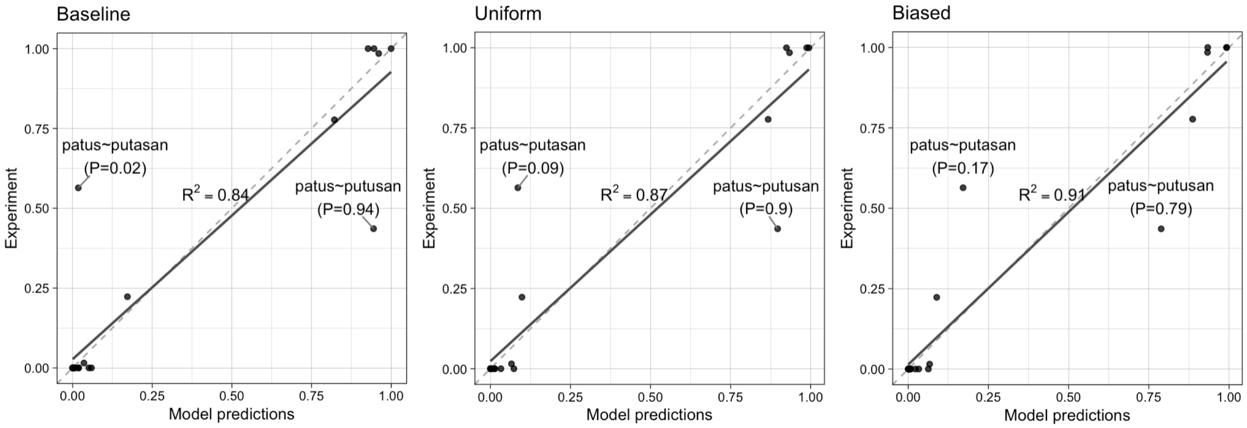


Figure 6: Predictions of models trained ont he lexicon, plotted against experimental results. Fitted regression lines are included.

Notably, the Biased model still under-predicts rates of [u]∼[a] alternation compared to experimental results. One possible reason for this is that [u]∼[a] alternations were initially extended by some Seediq speakers to a smaller degree, but that this pattern grew in magnitude over generations of speakers. This type of generational learning can be modeled in iterative models (e.g. Brighton, 2002; De Boer, 2000; Kirby, 2001; Ito & Feldman, 2022, and many more), and should be considered in future work.

## 7 Conclusion

Based on a survey of 341 Seediq verb paradigms, the current study finds that Seediq paradigms show a striking asymmetry, whereby the unsuffixed slots of the paradigm can be used to predict the suffixed forms with much higher accuracy than the other way around. This asymmetry was demonstrated with a morphological mapping model which uses the Minimal Generalization Learner (Albright, 2002b).

This asymmetry is expected if there has been a gradual restructuring of Seediq verb paradigms based on the unsuffixed forms. Asymmetric restructuring is unexpected in a composite UR approach, where reanalyses from all slots of a paradigm are possible. In contrast, it is the natural outcome under the single surface-based hypothesis, if Seediq speakers have selected an unsuffixed paradigm slot to be the base.

These results are supported by a production experiment, where speakers were found to extend generalizations about vowel alternations from the isolation stem, and productively apply a vowel matching pattern to gapped forms that have no listed suffixed forms. Interestingly, speakers also extended the vowel matching pattern to more forms than was predicted by a grammar that only learns morphological mappings based on statistical generalizations.

Based on these findings, I offer a stem-base grammar of Seediq morphophonology, where alternation is driven by phonological constraints rather than morphological correspondence. This model is implemented in MaxEnt, allowing it to account for gradient rates of alternation in Seediq. NUC-ID(V) is used to enforce post-tonic vowel alternation. Additionally, I propose that speakers' non-viridical extension of vowel matching is rooted in a complexity bias, which could be implemented in my MaxEnt model as a Gaussian prior.

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